NUMERICAL ANALYSIS OF HYDRAULIC TRANSIENTS IN PIPELINE SYSTEMS WITH VARYING BOUNDARY CONDITION USING LAPLACE TRANSFORMATIONS AND FREQUENCY DOMAIN METHOD [+]

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Abstract :

This paper foucs on the different between the analytical solution for the variation in hydraulic grade line (HGL). Basic fluid equations solved either, in the time domain, using classical method of charectaristics (MOC) and compare the results with Laplace transformations method and Frequency domain method. The main diffrence between these methods are, the Frequency domain method depends in the analytical solution on the Fourier series approach, but this approach is not well suiet to the case where the boundary conditions vary during the transient event. A Laplace transform solutoin approach overcomes this difficulity accordingly, the results for the pipeline system having varing demand showen that the Laplace transformation sense to wave pressure accur due to suddenly change in flow rate eather than Frequency domain method. By applying these method firstly on assumed network having suddenly chang in flowrate, then applied the mathematical model on Al-Razaza pumping station that suffers from transient flow due to suddenly change in pumping rate, and assumed this practical application to this study. Normalized hypebolic governing equations for apressur transient in a pipeline with change in demand are derived, where the discontinuity induced by a variations in demand by using a delta function. The effects of the change in demand on pipeline system transients induced by a pulse boundary perturbation and continuously changing boundary perturbation are invistigated in detail.

التحليل العددي للجريان الانتقالي في شبكات الانابيب ذات التصريف المتغير باستخدام تحويلات لابلاس وطريقة المدى الترددي

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المستخلص :

في هذا البحث تم ايضاح الفرق بين التحليل العددي لحالة التغير الحاصلة في خط الانحدار الهيدروليكي بسبب حدوث الجريان الانتقالي. استخدمت في التحليلال المعادلات الاساسية للهيدروليك حيث استخدمت الطريقة التقليدية التي تعتمد في تحليلها المدى الزمني والتي اكثرها شيوعا طريقة الخواص ومقارنة النتائج مع طريقتين الاولى تعتمد

[+] Received on 22/1/2013 , Accepted on 7/5/2014
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تحويلات لابلاس في التحليل والثانية طريقة المدى الترددي وتعتمد تحويلات فورير. تمت دراسة وتحليل موجة الضغط لشبكة مفترضة يتغير فيها التصريف بشكل مفاجئ ثم طبق الموديل الرياضي على محطة ضخ الرزازة التي تعاني من ظاهرة حدوث الجريان الانتقالي ايضا بسبب التغير الفجائي للتصريف واعتبرت تطبيق عملي حي للبحث. لوحظ ان الطريقة التي تعتمد تحويلات لابلاس اكثر تطابق وملائمة عندما تكون الشبكة ذات تصريف متغير من الطريقة التي تعتمد تحويلات فورير حيث ان الاخيرة لا تتحسس كثيرا بالتغيرالفجائي الحاصل للتصريف في الشبكة. تم معالجة المعادلات التي تحكم خاصية الجريان الانتقالي عندما تحصل زيادة مفاجئة للتصريف بسبب دخول مضخات اضافية على الشبكة رياضيا وذلك باستخدام دالة سميت دلتا. تاثيرات التغير في الطلب على منظومات الانابيب تم استنتاجها باستخدام موجات اضطراب محددة واضطرابات متغيرة بشكل متواصل وتم تحليلها بشكل تفصيلي.

1- Introduction :

Hydraulic transients are the time-varying phenomena that follow when the equilibrium of steady flow in a system is disturbed by a change of fiow that occurs over a relatively short time period. The pipe flow and pressure transients can be described by a set of non-linear hyperbolic equations derived from the conservation of mass and Newton's seconed law of motion (conservation of linear –momentom). A closed-form solution for these equations is impossible due to the non-linearity of the momentum equation. A number of methods have been developed to solve these equations analytically where the non-linear term is either neglected [1] or linearized [2,3], and numerically using the method of chacteristics (MOC) and other numerical methods [3,4]. Predominately, transient pipe flows are studied using one-dimensional models assuming a uniform velocity profile. The neglected two-dimensional or three dimensional effects are normally approximated by unsteady-friction models [5,6] with reasonable success.

2- Theoretical analysis :

2-1- Development of the Characteristic Equations :

Neglecting the spatial variation of (V) and (P) whenever both space- and time-varying terms appear in the same equation, in general, the spatial variations are much less significant in determining the solution behavior than the time-varying terms. Then we have two independent partial differential equations [7]

$$\frac{\partial V}{\partial t}+\frac{1}{\rho}\frac{\partial p}{\partial s}+g\frac{dz}{ds}+\frac{f}{2D}V|V|=0 \qquad \text{-- } 1$$

$$a^2\frac{\partial V}{\partial s}+\frac{1}{\rho}\frac{\partial p}{\partial t}=0 \qquad \text{-- } 2$$

Using l as a constant linear scale factor, sometimes called a Lagrange multiplier, one possible combination is

$$\lambda\left(\frac{\partial V}{\partial t}+\frac{1}{\rho}\frac{\partial p}{\partial s}+g\frac{dz}{ds}+\frac{f}{2D}V|V|\right)+\left(a^2\frac{\partial V}{\partial s}+\frac{1}{\rho}\frac{\partial p}{\partial t}\right)=0 \qquad \text{-------------------------- 3}$$

Regrouping terms,

$$\left(\lambda\frac{\partial V}{\partial t}+a^2\frac{\partial V}{\partial s}\right)+\left(\frac{1}{\rho}\frac{\partial p}{\partial t}+\frac{\lambda}{\rho}\frac{\partial p}{\partial s}\right)+\lambda g\frac{dz}{ds}+\frac{\lambda f}{2D}V|V|=0 \qquad \text{-------------------------- 4}$$

After restrictive conditions on the independent variables in each equations, the two partial differential equations replaced by two pair of ordinary differential equations, the new form of Eqs.(3, 4) is now

$$\frac{dV}{dt}-\frac{g}{a}\frac{dH}{dt}+\frac{f}{2D}V|V|=0 \quad \text{only when} \quad \frac{ds}{dt}=-a \qquad \text{----------------------------- 5}$$

$$\frac{dV}{dt}-\frac{g}{a}\frac{dH}{dt}+\frac{f}{2D}V|V|=0 \quad \text{only when} \quad \frac{ds}{dt}=-a \qquad \text{----------------------------- 6}$$

2-1-1 The Finite Difference Representation :

In computer manipulations, the Eqs.(5, 6) having new forms, in finite difference form, Eq.(8) becomes

$$\frac{V_P-V_{Le}}{t_P-0}+\frac{g}{a}\frac{H_P-H_{Le}}{t_P-0}+\frac{f}{2D}V_{Le}|V_{Le}|=0 \qquad \text{------------------------------ 7}$$

And Eq.(9) becomes

$$\frac{V_P-V_{Ri}}{t_P-0}-\frac{g}{a}\frac{H_P-H_{Ri}}{t_P-0}+\frac{f}{2D}V_{Ri}|V_{Ri}|=0 \qquad \text{------------------------------ 8}$$

The analysis will apply to more than the first time interval, accordingly the term (t_p -0) replaced with (Δt) then the new form gives

$$C^+:\ (V_P-V_{Le})+\frac{g}{a}(H_P-H_{Le})+\frac{f\Delta t}{2D}V_{Le}|V_{Le}|=0 \qquad \text{----------------------------- 9}$$

And

$$C^-:\ (V_P-V_{Ri})-\frac{g}{a}(H_P-H_{Ri})+\frac{f\Delta t}{2D}V_{Ri}|V_{Ri}|=0 \qquad \text{------------------------------ 10}$$

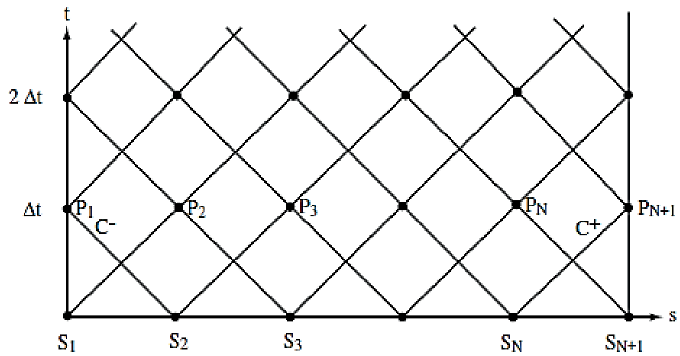

Figure (1): The characteristic grid for a single pipe

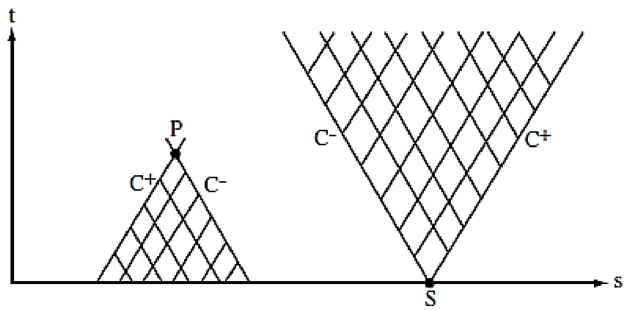

Figure (2): Disturbance propagation in the *s-t* plane

2-1-2 Setting Up the Numerical Procedure :

The values of (*H*) and (*V*) at the ends of the pipe were determined by using boundary conditions. Now we will arrange the solution procedure so it can be conveniently implemented on the computer.
By developing a pair of equations to find (H) and (V) at the interior points (points 2 through N). Solving Eqs.(9, 10) simultaneously to obtain

$$V_P = \frac{1}{2}\left[\left(V_{Le} + V_{Ri}\right) + \frac{g}{a}\left(H_{Le} - H_{Ri}\right) - \frac{f\Delta t}{2D}\left(V_{Le}\left|V_{Le}\right| + V_{Ri}\left|V_{Ri}\right|\right)\right] \quad \text{------------- } 11$$

$$H_P = \frac{1}{2}\left[\frac{a}{g}\left(V_{Le} - V_{Ri}\right) + \left(H_{Le} + H_{Ri}\right) - \frac{a}{g}\frac{f\Delta t}{2D}\left(V_{Le}\left|V_{Le}\right| - V_{Ri}\left|V_{Ri}\right|\right)\right] \quad \text{------------- } 12$$

2-1-3 Constant speed pump boundary condition (upstream end of pipe) :

This boundary condition offers the added complexity of having both (H_{P1} and V_{p1}) in the boundary condition. Consequently the boundary equation must be solved simultaneously with Eq.(12) to produce equations for ($H_{p1}$and V_{p1}) .
The pump boundary condition must be represents by simplest approach that is reasonably general is to represent the pump discharge characteristics by a quadratic equation of the form [7]

$$h_p = A_p{}' Q^2 + B_p{}' Q + C_p{}' \qquad \text{--------------------------------- 13}$$

In which (Q) is the pump discharge and(h_p) is the head increases across the pump. These variables are not identical to those in the (C^-) equation, so some adjustments on it by replace (Q) with (V_{P1}) and (h_p) with (H_{P1}– H_{sump}). Incorporating (H_{sump}) into ($C_P{}'$) and (A) into ($A_P{}'$) and ($B_P{}'$) lead to

$$H_{P_1} = A_p V_{P_1}^2 + B_p V_{P_1} + C_p \qquad \text{----------------------------------- 14}$$

Eq.(14) is solved simultaneously with the (C^-) characteristic equation, Eq.(12), the elimination of (H_{P1}) leads to the following equation for (V_{P1});

$$V_{P_1} - V_2 - \frac{g}{a}\left(A_p V_{P_1}^2 + B_p V_{P_1} + C_p\right) + \frac{g}{a} H_2 + \frac{f\Delta t}{2D} V_2 |V_2| = 0 \qquad \text{--------------------- 15}$$

Rearranging,

$$\left(\frac{g}{a} A_p\right) V_{P_1}^2 + \left(\frac{g}{a} B_p - 1\right) V_{P_1} + \left(V_2 + \frac{g}{a} C_p - \frac{g}{a} H_2 - \frac{f\Delta t}{2D} V_2 |V_2|\right) = 0 \qquad \text{---------------- 16}$$

This quadratic equation can now be solved for (V_{P1}). then a back substitution into Eq.(14) will yield (H_{P1}).

2-2 Solution Based on the Initial Conditions, Fourier Expansion Procedure :

By linearizing the governing equation for transients in a pipeline with varying demand, the equation having a form [2]

$$\frac{\partial^2 h^*}{\partial x^{*2}} = \frac{\partial^2 h^*}{\partial t^{*2}} + [2R + F_L \delta(x^* - x_L^*)] \frac{\partial h^*}{\partial t^*} \qquad \text{-------------------------------------- 17}$$

Where (x^*= x/L = dimensionless distance), t^*= t / (L/a) = the dimensionless time, L = pipeline length, a = wave speed, h^* = $(H – H_0) / H_1$ = the dimensionless head of the transient, H= transient head, H_0= steady state head, H_1= a reference head in a pipe, R= $fLQ_o/2DAa$

=resistance term, Q_o= steady state flow rate, f= Darcy-Weisbach factor, D= pipe diameter, A= pipe cross section area, $F_i= C_d A_L a/ A\sqrt{2Gh_{Lo}}$ = intake parameter, $C_d\, A_L$ =effective intake area, H_{Lo} =steady state head at the intake. $\delta(x^* - x_L^*)$ = Dirac delta function and $x_L^* = x_L/L$ = dimensionless intake function.

For the pipeline with constant head, the boundary conditions are given by

$$h^*(0,t^*)=0 \text{ and } h^*(1,t^*)=0 \qquad \text{------------------------ 18}$$

And the initial conditions of the pipeline transients may be defined as

$$h^*(x^*,\ 0)=a_{IC}(x^*) \text{ and } \frac{\partial h^*(x^*,0)}{\partial t^*}=b_{IC}(x^*) \qquad \text{------------------------- 19}$$

Is the flow out induced damping factor for harmonic component *(n)*.the Fourier coefficients, A_n and B_n, are

$$A_n = 2\int_0^1 a_{IC}(x^*)\sin(n\pi x^*)dx^* \qquad \text{------------------------ 20}$$

$$B_n=\frac{2}{n\pi}\int_0^1 b_{IC}(x^*)\sin(n\pi x^*)dx^*+\frac{(R+R_{nL})A_n}{n\pi} \qquad \text{------------------------ 21}$$

2-3 Solution for time dependent boundary conditions, Laplace transformation procedure :

The solution is based on known initial conditions for the pipeline transient. However, it is more practical to measure the time varying initiation process rather than measure the transient distribution along a pipeline. Due to the limitation of the separation of variables technique in solving partial differential equations with time dependent boundary conditions, a solution considering the time dependent initiation process is given using the Laplace transform method [3].

If a pipeline transient is initiated from a steady state condition by a downstream perturbation process, the governing equation and the corresponding boundary and initial conditions are

$$\text{B.C.} \qquad h^*(0,t^*)=0 \text{ and } h^*(1,t^*)=f(t^*) \qquad \text{--------------------------- 22}$$

$$\text{I.C.} \qquad h^*(x^*,0)=0 \text{ and } \frac{\partial h^*(x^*,0)}{\partial t^*}=0 \qquad \text{-------------------------- 23}$$

In which [$\boldsymbol{f(t^*)}$] = dimensionless head at the downstream end of the pipeline.

3-3-1 Pipeline without change in demand discharge :

For a pipeline without any change in demand, F_L= 0, and applying Laplace transforms to gives

$$\frac{\partial^2 \tilde{H}(x^*,s)}{\partial x^{*2}} = [s^2 \tilde{H}(x^*,s) - sh^*(x^*,0) - \frac{\partial h^*(x^*,0)}{\partial t^*}] + 2R[s\tilde{H}(x^*,s) - h^*(x^*,0)] \quad \text{--------------------- 24}$$

Considering the conditions in Eqs.(22, 23), is expressed as

$$\frac{\partial^2 \tilde{H}(x^*,s)}{\partial x^{*2}} = s^2 \tilde{H}(x^*,s) + 2Rs\tilde{H}(x^*,s) \quad \text{----------------------- 25}$$

Applying the Laplace transforms in Eq.(25) gives

$$\tilde{H}(0,s) = 0 \text{ , and } \tilde{H}(1,s) = \tilde{F}(s) \quad \text{------------------------26}$$

In which $\tilde{F}(s) = L\{ f(t^*) \}$ = Laplace transform of $f(t^*)$.
For a frictionless pipe, R= 0, and the solution for Eq.(26) is

$$\tilde{H}(x^*,s) = C_1 e^{sx^*} + C_2 e^{-sx^*} \quad \text{--------------------------- 27}$$

Substituting Eq.(26) in Eq.(27) and solving gives

$$C_1 = \frac{\tilde{F}(s)}{e^s - e^{-s}} \text{ , and } C_2 = \frac{-\tilde{F}(s)}{e^s - e^{-s}} \quad \text{------------------------ 28}$$

Therefore, Eq.(27) can be expressed as

$$\tilde{H}(x^*,s) = \sum_{n=0}^{\infty} [\tilde{F}(s) e^{-s(2n+1-x^*)} - \tilde{F}(s) e^{-s(2n+1+x^*)}] \quad \text{------------------- 29}$$

2-3-2 Pipeline with varying demand discharge :

For a pipeline with varying discharge, the pipeline can be considered as two portions divided by the intakes that gives a new demand discharge with a small neighborhood (2ε) as shown in Fig.(3)[3]

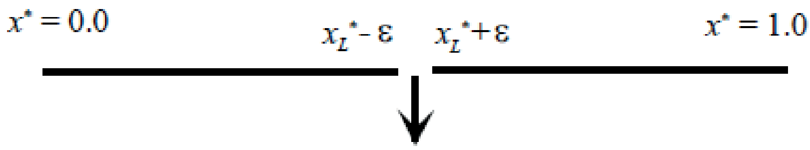

Figure (3): A pipeline with varying discharge

Integrating Eq.(25) over a small neighborhood on either side of the leak gives

$$\int_{x_L^*-\varepsilon}^{x_L^*+\varepsilon} \frac{\partial^2 h^*}{\partial x^{*2}} dx^* = \int_{x_L^*-\varepsilon}^{x_L^*+\varepsilon} \left(\frac{\partial^2 h^*}{\partial t^{*2}} + 2R\frac{\partial h^*}{\partial t^*}\right) dx^* + \int_{x_L^*-\varepsilon}^{x_L^*+\varepsilon} F_L \frac{\partial h^*}{\partial t^*} \delta(x^* - x_L^*) dx^* \qquad \text{------------------------- } 30$$

Letting (ε) approach zero, the first integral on the right hand side of Eq.(30) is zero. Thus Eq.(30) becomes

$$\left.\frac{\partial h^*}{\partial x^*}\right|_{x_L^* - \varepsilon}^{x_L^* + \varepsilon} = F_L \left.\frac{\partial h^*(x^*, t)}{\partial t^*}\right|_{x^* = x_L^*} \qquad \text{------------------------- } 31$$

The governing equations for the two frictionless pipe sections on either side of the varying discharge are

$$\frac{\partial^2 h_1^*}{\partial x^{*2}} = \frac{\partial^2 h_1^*}{\partial t^{*2}} \qquad (0 \le x^* < x_L^*,\ t^* > 0)$$

$$\frac{\partial^2 h_2^*}{\partial x^{*2}} = \frac{\partial^2 h_2^*}{\partial t^{*2}} \qquad (x_L^* < x^* \le 1,\ t^* > 0) \qquad \text{----------------------- } 32$$

Fig.(4) show the flowchart for surge phenomenon in water distribution systems.

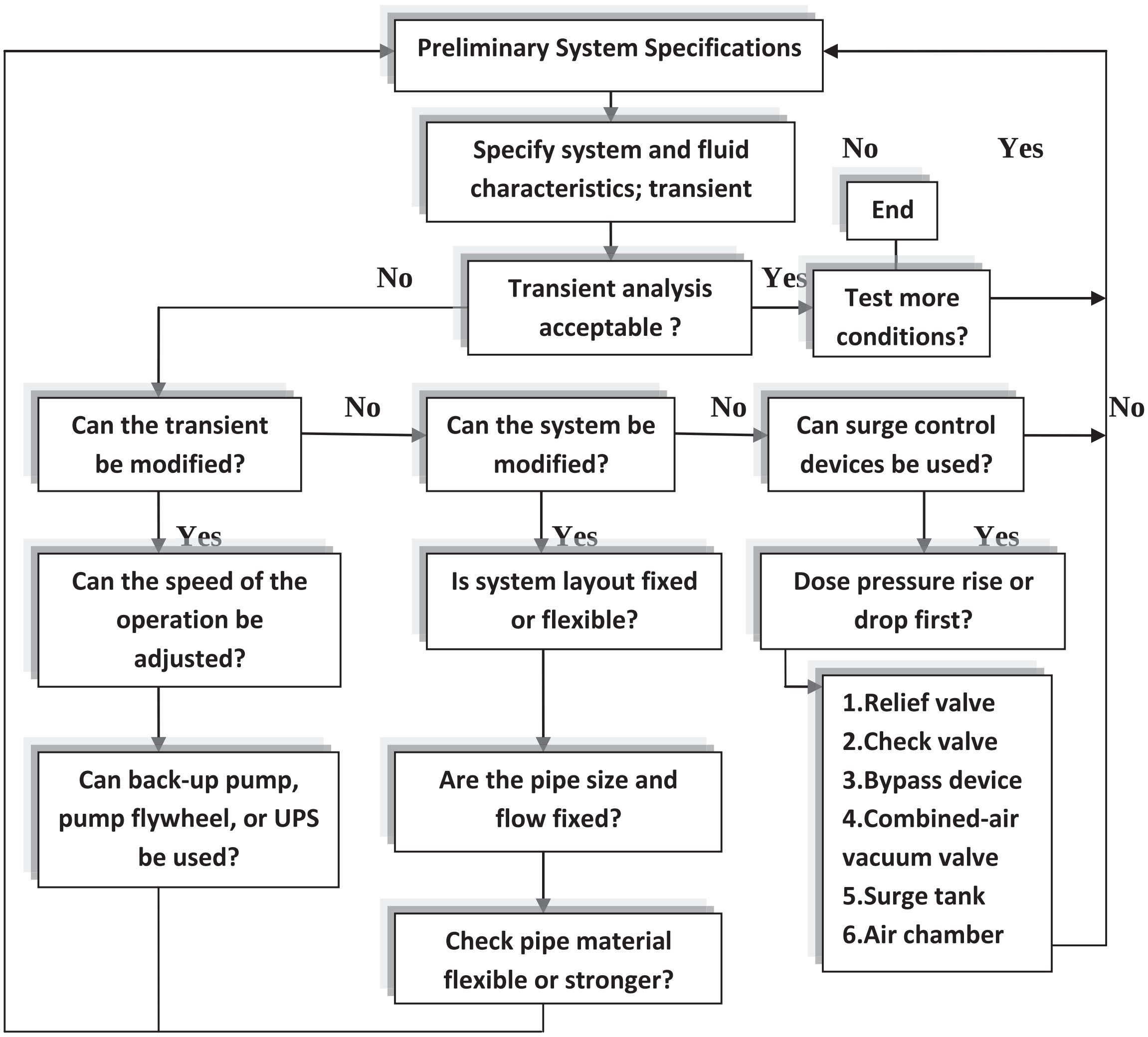

Figure (4): flow chart for transient flow analysis in pipeline systems

3- <u>Hydraulic Applications :</u>

- **Theoretical case study**

The first case study shown in Fig.(5), the simple pipeline system consists from (6) pipes with characteristics shown in table (1), and (6) nodes all data about them shown in table (2), the pump is incorporated into a network and located at one of three reservoirs. The analysis assumes the demand discharge is suddenly increase from (50 gpm) to (790 gpm) due to pump on pipe (6) restart to operate at (20 sec.) and the transient occur downstream the pump, the wave speed is 3000 ft. /s and friction factor (0.02) for all pipes, [7].

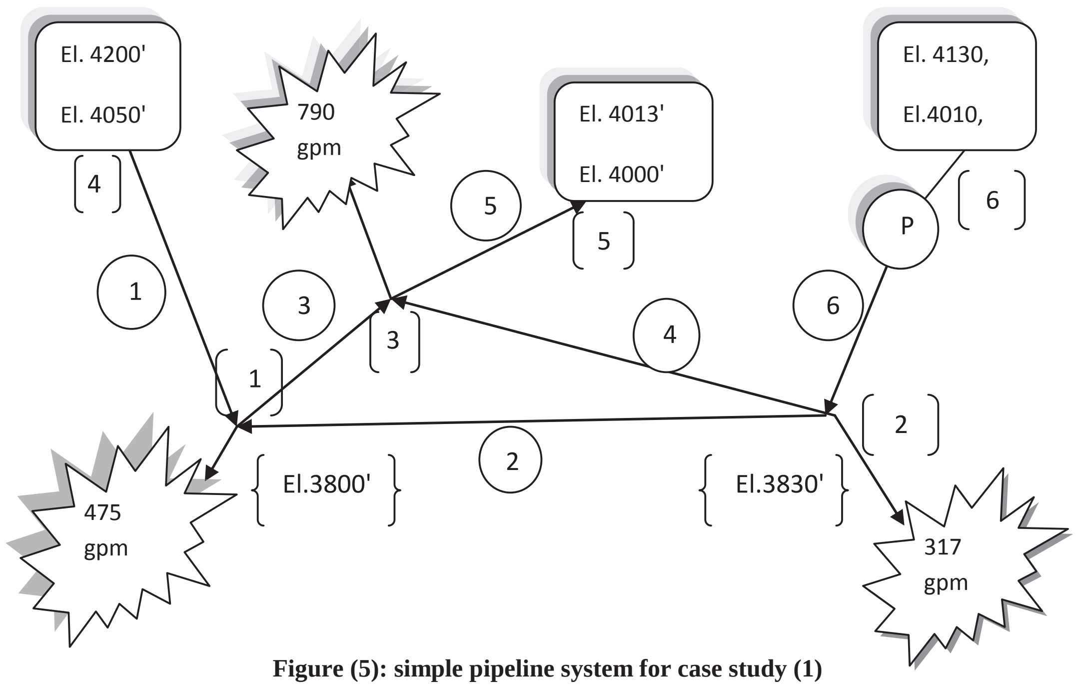

Figure (5): simple pipeline system for case study (1)

Table (1): pipeline data for case study (1)

Pipe No.	Nodes From	to	Length Ft.	Dia. In.	*f*	Q gpm	Vel. Ft/s	H_L ft.
1	4	1	3300	12.0	0.02	340.1	0.96	0.40
2	2	1	8200	8.0	0.02	273.0	1.47	1.91
3	1	3	3300	8.0	0.02	138.1	0.88	0.54
4	2	3	4900	12.0	0.02	1110.0	3.15	3.57
5	3	5	3300	6.0	0.02	458.1	5.20	20.27
6	6	2	2600	14.0	0.02	1700.0	3.54	3.71

Table (2): nodes data for case study (1)

Node	Demand gpm	El. Ft.	Head Ft.	Pressure Psi.	HGL El. Ft.
1	475	3800	398.68	172.76	4198.68
2	317	3830	384.38	166.57	4214.38
3	790	3770	426.89	184.99	4196.89
6	-1700	4010	214.03	92.74	4224.03
4	-340	4050	150.00	65.00	4200.00
5	458	4000	130.00	56.33	4130.00

- **Practical application**

This case study focus on the transient analysis for Al-Razaza pumping station, this station suffering a transient caused by suddenly change in discharge due to operate PS_2 on the same main pipes for PS_1. The original design for this pumping station contains only PS_1 having (5 axial pumps), (4 pumps in service and 1 pump stand by) connects (2 main pipes) with (2 m) diameter and (800 m) long (2 pumps) for each individual pipe, the characteristics for these pumps;[10]

Discharge (Q) = 3.8 m^3/s
Revolution (rpm) =423 rpm
Head (H) =23 m
Power (HP) for the electric motor = 1600 Kw

Due to increasing in the drainage water for Al-Razaza drain, these pumps be not sufficient to occupies this drain especially in winter and spring, this problem being need to solve, the staff in this station installed PS_2 in parallel to the PS_1. The new station contains (12 horizontal pumps), 10 pumps operate and 2 pumps stand by, each 6 pumps connects with collector pipe, each one goes to main pipe then to Al-Razaza lack. The characteristics for these pumps;

Discharge (Q) = 1 m^3/s
Revolution (rpm) = 1200 rpm
Head (H) =21 m
Power (HP) = 340 Kw

When the drainage water rises, PS_2 be operated, here the transient flow start soon and 2 stand pipes having (20 m) height and (3 m) diameter installed one on each of the main pipe at distance (20 m) from axial pumps and (23 m) from horizontal pumps, these stand pipes flooded. The station was tested when this phenomenon being start and the layout suffering transient, the nodal was tested in the downstream of the stand pipes (1). Fig.(6) shown the layout for the whole station, table (3) give all hydraulic pressures for the main pipes when three axial pumps in service with variable numbers of horizontal pumps, the friction factor for the main pipes and collector pipes (0.031), the total rotary moment of inertia of each pump and motor unit is (475 $Ib\text{-}ft^2$), the wave speed for steel pipe (3490 ft/s).

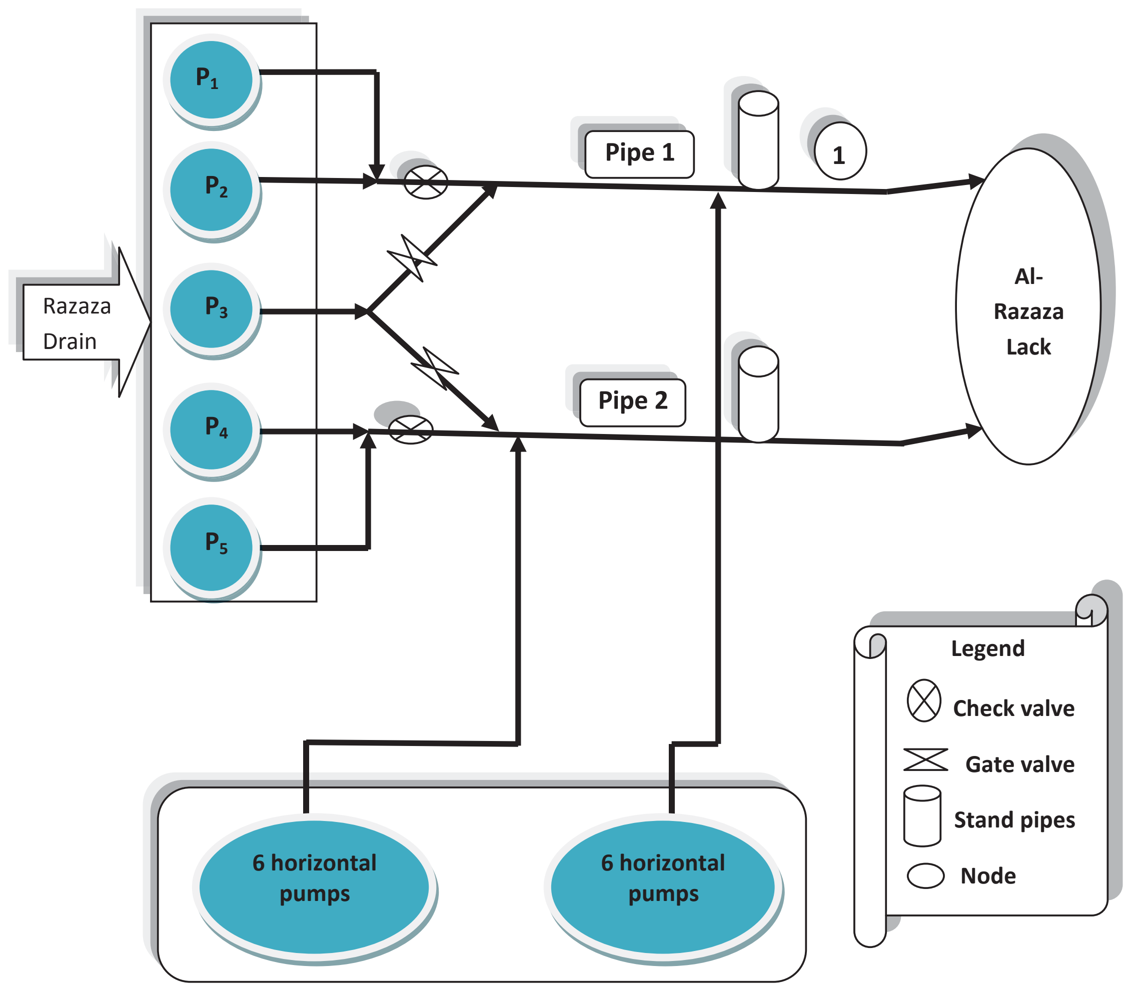

Figure (6): typical layout for Al-Razaza pumping station

Table (3): hydraulic pressures in the main pipes when three axial pumps in the service with variable numbers of horizontal pimps.

No. of horizontal pumps	Pressure on surge tank in pipe 1(m)	Pressure on surge tank in pipe 2(m)	Velocity in pipe 1 (m/s)	Velocity in pipe (m/s)	Discharge in pipe 1(m^3/s)	Discharge in pipe 2(m^3/s)	H_L(m)in1	H_L in 2(m)
1	44.13	44.17	2.20	2.22	6.98	6.97	2.07	2.1
2	44.35	44.35	2.34	2.34	7.35	7.36	2.33	2.3
3	44.57	44.61	2.46	2.48	7.72	7.78	2.54	2.6
4	44.82	44.83	2.59	2.59	8.13	8.14	2.79	2.8
5	45.04	45.08	2.69	2.69	8.45	8.51	3.01	3.0
6	45.28	45.29	2.81	2.81	8.82	8.83	3.25	3.3
7	45.49	45.53	2.90	2.92	9.11	9.17	3.46	3.5
8	45.72	45.73	3.00	3.01	9.43	9.45	3.68	3.7
9	45.91	45.95	3.09	3.10	9.69	9.75	3.78	3.9
10	46.13	46.14	3.18	3.18	9.98	10.01	4.09	4.1

4- Conclusion and Recommendations :

During the observations of the analysis of the three methods used in this research, the method in which depend Laplace transforms in the analysis more accurate in sensing the pressure wave when suddenly change in demand or in other word in pumping rate in network systems. Fig.(7) clearly shown the wave pressure accurse in the assumed network after the suddenly change in flow rate and how the Laplace transforms gives the pressure wave more accurate from the others methods. The same results were obtained when Appling the model on Al-Razaza pumping station that suffering transient flow due to suddenly change in pumping rate, fig.(8) clearly shown this phenomenon.

The recommendations of the research can be summarized as follows

- Analysis of other cases of transient flow, such as more complex networks suffering transients due to air entrainment on fluid.
- Analysis transients due to changes in valve settings, or change within the transmission system, change in storage tank operation.
- Analysis of other network systems having possibility of sudden changes in pressure due to change in temperature or sudden move of hydraulic ram.

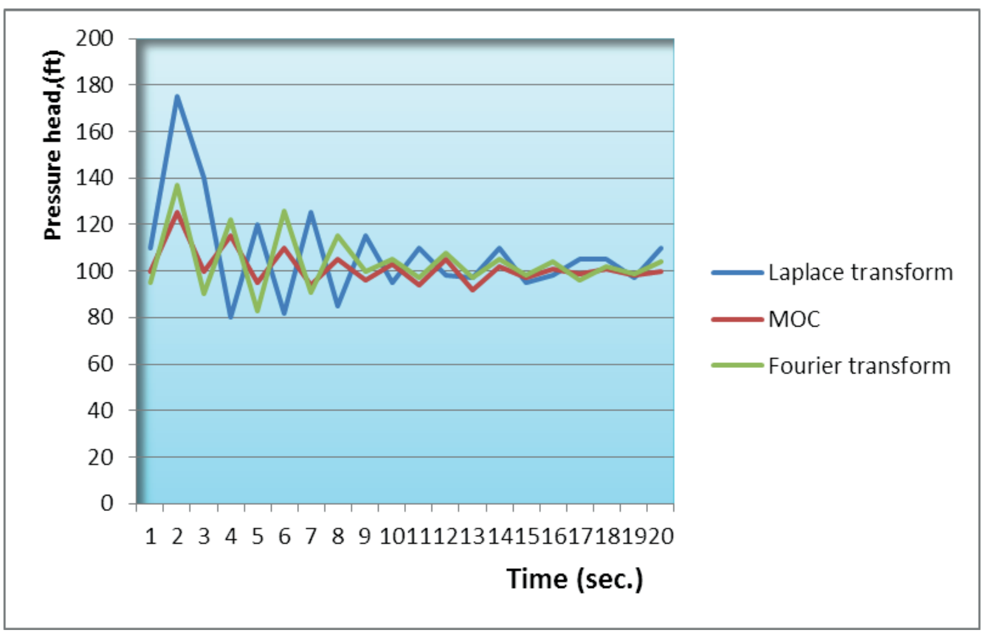

Figure (7): pressure head downstream the pump for case study(1) to all methods.

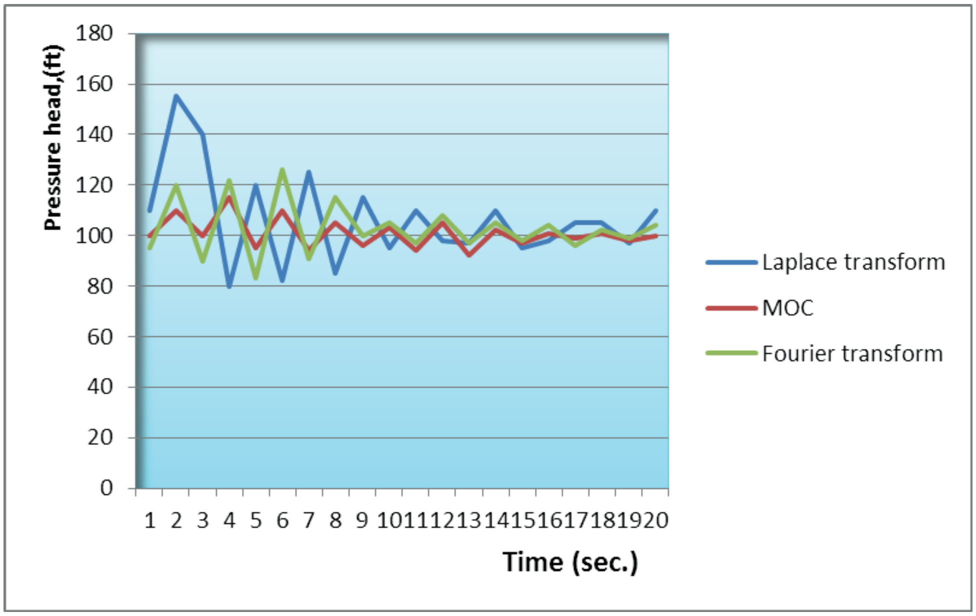

Figure (8), Pressure head down stream of the stand pipe at point (1) on main pipe (1) for Al-Razaza pumping station to all methods.

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Nomenclature :

Symbol	Quantity	Unit SI	Unit English	Dimension
A	Cross section area of pipe	m^2	ft^2	$\mathbf{L^2}$
A_n	Fourier coefficients		Dimensionless	
A_P	Polynomial constants		Dimensionless	
a	Wave celerity	m/s	ft/s	**L/t**
$a_{IC}(x^*)$ and $b_{IC}(x^*)$	Piecewise continuous functions		Dimensionless	
$C_d A_L$	Effective intake area	m^2	ft^2	$\mathbf{L^2}$
D	Pipe diameter	m	ft	**L**
$F_i = C_d A_L a / A\sqrt{2Gh_{Lo}}$	Intake parameter		Dimensionless	
f	Friction factor		Dimensionless	
g	Gravitational acceleration	m/s^2	ft/s^2	$\mathbf{L/t^2}$
H	Transient head	m	ft	**L**
H_0	Steady state head	m	ft	**L**
H_1	Reference head in a pipe	m	ft	**L**
H_{Lo}	Steady state head at the intake	m	ft	**L**
h_p	Pump head	m	ft	**L**
ℓ	Scale factor		Dimensionless	
P	Pressure	Pa	Ib/ft^2	$\mathbf{ML^{-1}t^{-2}}$
Q_o	Steady state flow rate	m^3/s	ft^3/s	$\mathbf{L^3t^{-1}}$
$R = fLQ_o/2DAa$	Resistance term		Dimensionless	
s	Space	m	ft	**L**
$t^* = t/(L/a)$	Dimensionless time		Dimensionless	
$x^* = x/L$	Dimensionless distance		Dimensionless	
V	Average velocity	m/s	ft/s	$\mathbf{Lt^{-1}}$
$x_L^* = x_L/L$	Dimensionless intake function		Dimensionless	
ρ	Mass density	Kg/m^3	Ib/ft^3	$\mathbf{ML^{-3}}$
λ	Lagrange multiplier		Dimensionless	
sub scribt$)_P$	Node value for finite deference			
sub scribt$)_{Le}$	Left value for finite deference			
sub scribt$)_{Ri}$:	Right value for finite deference			
$\delta(x^* - x_L)^*$	Dirac delta function		Dimensionless	
ε	small neighborhood	m	ft	**L**