U.P.B. Sci. Bull., Series D, Vol. 80, Iss. 1, 2018 ISSN 1454-2358

DYNAMIC FRICTION IN COULOMBIAN RUBBER DAMPER AT LOW VELOCITIES

Haider WAHAD[1, 2], Andrei TUDOR[3], Kussay SUBHI[1]

This paper describes a study of the dynamic friction between the rubber and steel ball at low sliding velocities. The highest velocity did not exceed a few millimeters per minute so that frictional heating was negligible. The results show that the friction decreases with the sliding velocity to a minimum value. Therefore, that friction appears from normal force and sliding velocity, and that both are directly related to the viscoelastic properties of the rubber. In the sliding contact of rubber (viscoelastic material) on a rigid substrate, the coefficient of friction may depend on normal force, sliding velocity. The friction is characterized by a Striebeck curve type (friction coefficient μ versus sliding speed v). Theoretical model and experimental results of friction coefficient between the rubber and steel ball on sliding velocities for different values is investigated.

Keywords: sliding velocity, rubber, coefficient of friction, normal load.

1. Introduction

Friction exists everywhere in both nature and man-made objects. It can be beneficial, such as book flipping or product transportation on conveyor system [1,2]. Sometimes friction is expected to be reduced because it causes the unwanted loss of energy [3, 4, 5]. A simple but today still widely used law of friction is Amontons' law which states that friction force is proportional to the normal force, the ratio is known as coefficient of friction. However, since Coulomb it has been already known that coefficient of friction may depend also on material, normal load, system size, time, sliding velocity [6, 7]. It is well known that for rubber like viscoelastic materials friction in a contact with a hard counter surface is velocity-dependent [8]. A large number of studies have shown that friction between viscoelastic material and hard surface depends on loading parameters and material parameters, in particular strongly on sliding velocity, normal load and temperature [9, 10]. The phenomena of friction between rubber

[1] Ph.D. Student, faculty of Mechanical Engineering and Mechatronics, Dept. of Machine Elements and Tribology, University POLITEHNICA of Bucharest, Romania
e-mail: haider.shahad@yahoo.com
[2] Eng., Ministry of Science and Technology, Baghdad, Iraq
[3] Prof., Dept Machine Elements, and Tribology, POLITEHNICA of Bucharest, Romania, email: andreitudor17@gmail.com

and rigid materials have many applications. The rubber has elasticity makes it suitable for various kinds of shock absorbers and for specialized machinery mountings designed to reduce vibration.

The aim of this work is to determine the friction force and friction coefficient for the rubber against steel ball at very low sliding velocities.

2. Theoretical Model of the Friction

In spite of the obvious practical importance, recently it was argued that, although there is a large number of parameters which affect the friction, the number of parameters can have reduced by choosing corresponding parameter combinations which mostly directly and robustly determine friction, lubricant viscosity (η), and coefficient of friction (μ_0), dependent on the lubricant rheology.
In the area of low sliding speed, at zero velocity, there is static friction characterized by static coefficient of friction μ_s, which is dependent on the real contact pressure, couple materials [11]. The coefficient of friction is maintained constant until a certain velocity v_0:

$$\mu_o = \mu_s \ for \ \ 0 \le v \le v_o \, . \quad (1)$$

When the sliding velocity increases over v_0 limit, the normal force is transmitted through the real area, in this case the friction coefficient decreases curvilinear until it reaches a limit value μ_m [12]. It is accepted in the zone of limit, mixed and dry friction a parabolic variation as:

$$\mu = av^2 + bv + c \quad for \ v_o < v < v_{cr} \, , \quad (2)$$

where v_{cr} is the critical velocity and a, b, c are constants for certain known points (v_0, μ_S), (v_m, μ_m) minimum point.

For velocities higher than v_m, a fluid friction begins. In this case the friction coefficient having an analytical form:

$$\mu = c_h v_a + \mu_o \quad for \ v_a \ge v_{cr} \, . \quad (3)$$

The following notations are made: $\mu_a=\mu/\mu_s$; $\mu_{ma}=\mu_m/\mu_s$; $v_{am}=v/v_m$; $v_{0m}=v_0/v_m$; $v_{0a}=v_0/v_m$; $v_{a\,cr}=v_{cr}/v_m$; $c_{ha}=c_h\, v_m/\mu_s$.

Putting the continuity conditions everywhere and derivability in the zone of dry, limit and mixed of friction coefficient function, Striebeck type, the constants a, b, c, μ_0 can be determined.
Thus, the relative friction-sliding coefficient has the expression

$$\mu_a = \begin{cases} \mu_s = \mu_o & if \ o \le v_{am} \le v_{om} \\ a_a v_{am}^2 + b_a v_{am} + c_a & if \ v_{om} < v_{am} < v_{acr} \, . \\ c_{ha} v_{am} + \mu_o & if \ v_{am} \ge v_{cr} \end{cases} \quad (4)$$

The Striebeck curves depend on the minimum value of the friction coefficient μ_{ma}, lubricant viscosity (c_{ha}), and minimum sliding velocity (v_{om}). The Fig. 1 shows the theoretical curve for minimum values of the friction coefficient.

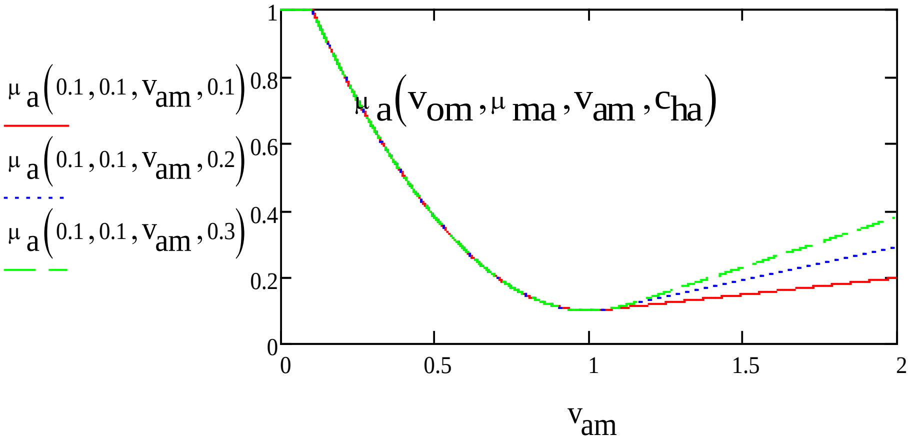
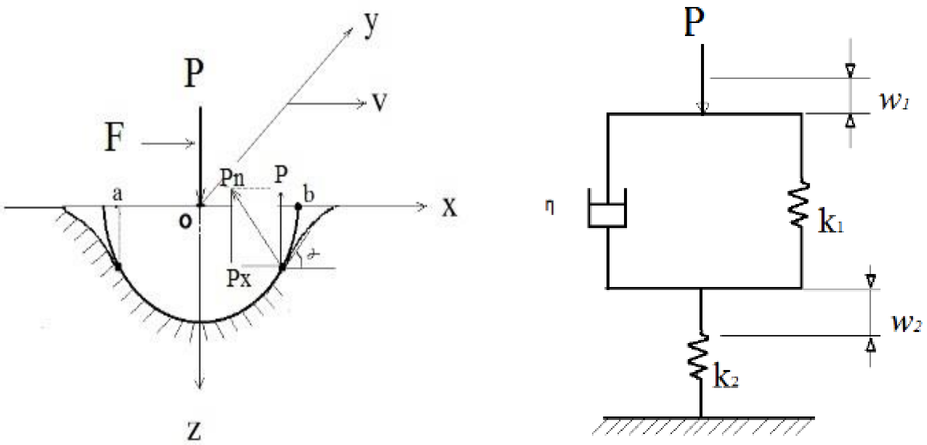


Fig. 1 Striebeck curves for some minimum friction coefficients

The rubber can be approximated by a Standard solid type, characterized by local stiffness k_1 and k_2 and the local parameter of viscosity η as shown in Fig. 2,b.When considering the case of the steady motion for the rubber medium base in the coordinates system *Oxyz* (Fig.2,a).



(a) (b)

Fig. 2 The forces system (a) and Standard solid model (b)

The constitutive equations of the Standard solid model are

$$\eta \frac{dw_{1\xi}}{dt} + k_1 w_{1\xi}(t) = p_\xi(t);\; k_2 w_{2\xi}(t) = p_\xi(t)\,, \tag{5}$$

where w_1 and w_2 are the material displacements in a point (ξ) of the viscoelastic zone (viscosity η and spring k_1) and the elastic zone (spring k_2), t the time, p_ξ the contact pressure in the point (ξ) [14].

In the direction of sliding with velocity V, the time of each point in contact surface is $t = (b-x)/V$. Thus, the constitutive equation (5) takes the form as in equation (6):

$$w_1(x) = w_{1\xi}\left(\frac{(b-x)}{V}\right);\ w_2(x) = w_{2\xi}\left(\frac{(b-x)}{V}\right), \tag{6}$$

$$-\eta\frac{dw_1}{dx} + k_1\, w_1(x) = p(x);\ k_2\, w_2(x) = p(x)\,. \tag{7}$$

The conditions that the punch moves regularly and progressively, that it is static in the plane Oxy, are the known conditions of the equilibrium: the equality to the zero of the forces sum affecting the punch (F, P, the base response) and the equality to the zero of the sum of their moments with respect to an arbitrary point on the plane Oxy [13].The interaction between moving rigid punch and the fixed viscoelastic base is manifested by the external normal forces P and tangential F (Fig. 2,a). In y- direction (perpendicular to the plane xOz) the forces P and F are considered uniformly distributed. Under the constant speed of the punch, the tangential force F is the friction force component. The profile of spherical shape is

$$g(x,y) = (x^2 + y^2)/2R\ , \tag{8}$$

where x , y are coordinates of contact point and R is the radius of ball is shown in Fig. 2,a. At any point (x) on the profile of the punch the $p(x)$ radial pressure acts with the two components [14].

$$p(x) = p_n\,(x)\cos\ \alpha(x); \tau\,(x) = p_n\,(x)\sin\alpha(x) = p(x)\tan\alpha(x)\,. \tag{9}$$

Taking into account the angle $\alpha(x)$ defined by the derivative of the function of the profile $\dot{g}(x) = tan\,\alpha$, result of the tangential tension on the sliding directions is

$$\tau(x) = p_x\,(x) = p_n(x).\,\dot{g}(x). \tag{10}$$

Thus, the two forces P and F are the results of the pressures in the direction z respective to x.

$$P = \int_{-a}^{b} p(x)dx\,; \tag{11a}$$

$$F = \int_{-a}^{b}\tau(x)dx = \int_{-a}^{b}\dot{g}(x)\,p(x)dx, \tag{11b}$$

under the assumption of a uniform movement, the tangential force is a deformation friction force (hysteresis component) and the deformation component of friction coefficient (μ_h) between the viscoelastic material with a rigid material.

$$\mu_h = \frac{F}{P}\ ; \tag{12a}$$

therefore

$$\mu_h = \frac{F}{P} = \frac{\int_{-a}^{b} \dot{g}(x)}{\int_{-a}^{b} p(} \qquad (12b)$$

To analyze the friction coefficient between rigid ball and band rubber it is necessary to use the dimenisionless parameters:

- The dimensionless sliding velocity is

$$V_a = \frac{\eta V}{k_1 R}\ , \qquad (13)$$

where V is the sliding velocity; R is the radius of the ball; k_1 is the rigidity of the rubber in the Standard solid model;

- The dimenisionless coordinates of boundary points of the contact are

$$b_a = \frac{b}{R}\ , \qquad a_a = \frac{a}{R}\ ;$$

- The dimensionless depth is $\delta_a = \delta/R$;
- The coordinates of any point in the contact area are $x_a = x/R,\ y_a = y/R$;
- The ratio of local rigidity of rubber is $r = k_2/k_1$;
- The parameter of velocity displacement of rubber is $\rho_a = \eta\ \rho$ and $\rho = V/c_n$, where c_n is the stiffness of the rubber in the normal direction of the contact surface.

The second order linear differential equation of contact pressure is

$$\frac{d^2 p_a}{dx_a^2} + C_1 \frac{dp_a}{dx_a} + C_o p_a = -r\left(1 + \frac{x_a}{V_a}\right), \qquad (14)$$

where

$$C_1 = \frac{r\rho_a + r + 1}{V_a}, C_o = \frac{r\rho_a}{V_a^2}, \qquad (15)$$

where $p_a = p/k_1 R$ is dimensionaless contact pressure.

The solution of this differential equation is determined on the basis of the characteristic equation and the particular solution:

$$\lambda^2 + C_1\lambda + C_o = 0\ , \qquad (16)$$

$$p_{oa} = d_1(\exp(\lambda_1 x_a)) + d_2(\exp(\lambda_2 x_a)), \qquad (17)$$

when λ_1, λ_2 are solutions of the equation (16), and d_1, d_2 are constants that are determined from the boundary conditions. The particular solution will be

$$p_{pa} = \frac{-r}{C_o}\left(\frac{x_a}{V_a} + 1 - \frac{rC_1}{C_o V_a}\right), \qquad (18)$$

thus, dimensionless pressure has expression

$$p_{at} = p_{oa} + p_{pa}. \qquad (19)$$

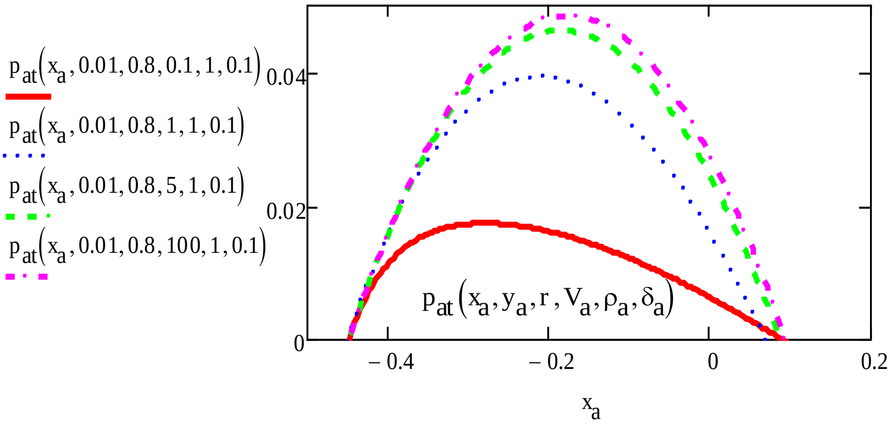


Fig. 3 The variation of dimensionaless pressure for different values of the sliding velocity

In the Fig. 3 shows the dimenssionless pressure values at some values of velocity parameter. We note that the maximum contact pressure value depends on the sliding velocity parameter.

The dimensionaless normal load is defined by expression

$$P_{nr} = \frac{P}{k_1 R^3}, \tag{20}$$

where P is the normal force.

The hysterezis coefficient of friction in equation (12) is shown in Fig. 4 and 5. The hysterezie coefficient of friction depends on the dimensionaless normal load P_{nr} and velocity displacement parameter ρ_a.

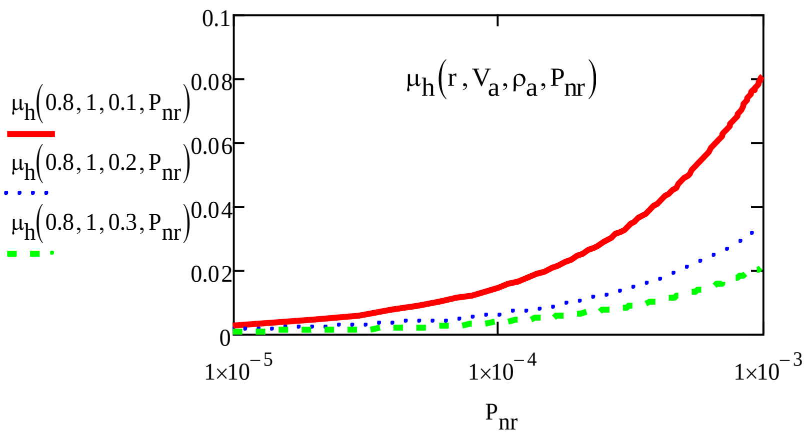


Fig. 4 The hysterezis coefficient of friction vs. the normal load

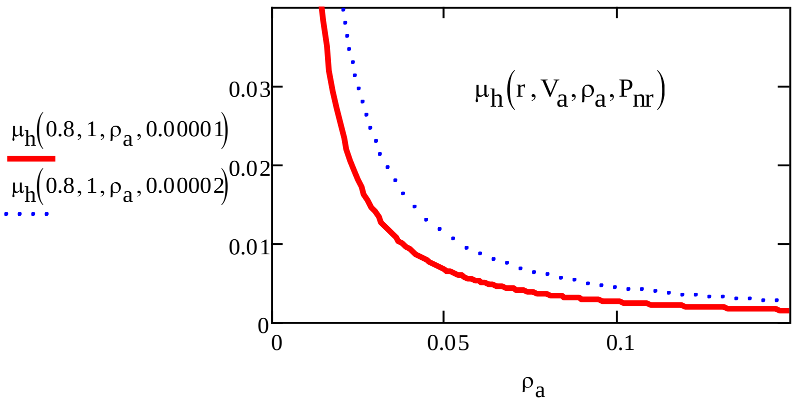


Fig. 5 The hysterezis coefficient of friction vs.velocity displacement parameter (b)

In Fig. 4 shows the hysteresis friction coefficient values increase when the normal loads increase and in Fig. 5 the coefficient of friction decreases when velocity parameter increases. The contact between rubber and rigid materials depends on the properties of the materials and the adhesion properties. The adhesion component of the friction force (F_{fa}) is determined on the basis of the contact area (A_c) and the adhesion stress (τ_f) between the rubber and the steel ball. The contact area is appreciated by the area of ball calotte of radius R and height δ. This height is the penetration of the ball in the viscoelastic material as a result of the normal force P. From the mechanical equilibrium condition of the pressure on the contact area (normal force P), the penetration $\delta_a = \delta/R$ is determined. For example, in Fig. 6 is shown the penetration dependence of the dimensionless load parameter P_{nr} with the pressure distribution of the equation (19). These dependences are obtained by Matchad 2000 program.

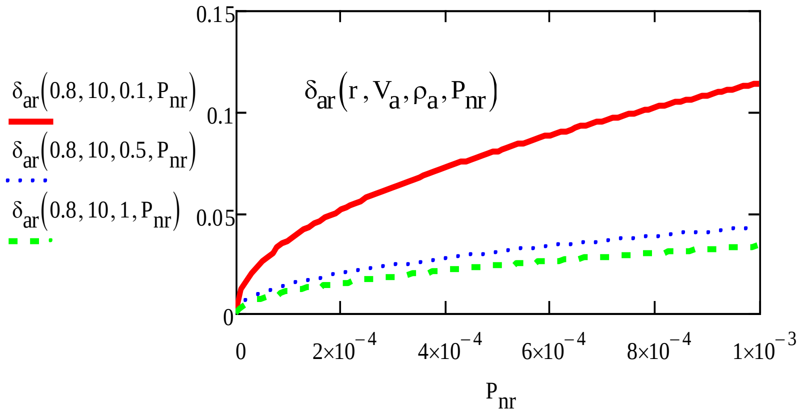


Fig. 6 The penetration of the rigid ball in the viscoelastic material

The adhesion component of friction is

$$F_{fa} = A_c \tau_f = 2\pi R\delta\tau_f \,, \tag{21}$$

and the adhesion component of friction coefficient has expression

$$\mu_a = \frac{F_{fa}}{P} = \frac{A_c\tau_f}{P} = \frac{2\pi R^2\tau_f\delta_a}{P} \,. \tag{22}$$

The total coefficient of friction is

$$\mu_t = \mu_h + \mu_a = \frac{\int_{-a}^{b} \dot{g}(x)p(x)dx}{\int_{-a}^{b} p(x)dx} + \frac{2\pi R^2\tau_f\delta_a}{P} \,. \tag{23}$$

The dimensionless shear strength of rubber is $\tau_{af} = \frac{\tau_f}{k_1 R}$.

Fig. 7 shows the values of friction coefficient of adhesion component with the normal load for two dimensionless velocities parameter ρ_a.

We assumed (ρ_a = 1 and ρ_a = 2), rigidity ratio r = 0.8 and dimensionless shear resistance τ_{af} = 0.003. We observe the friction coefficient decreases when the normal load increases.

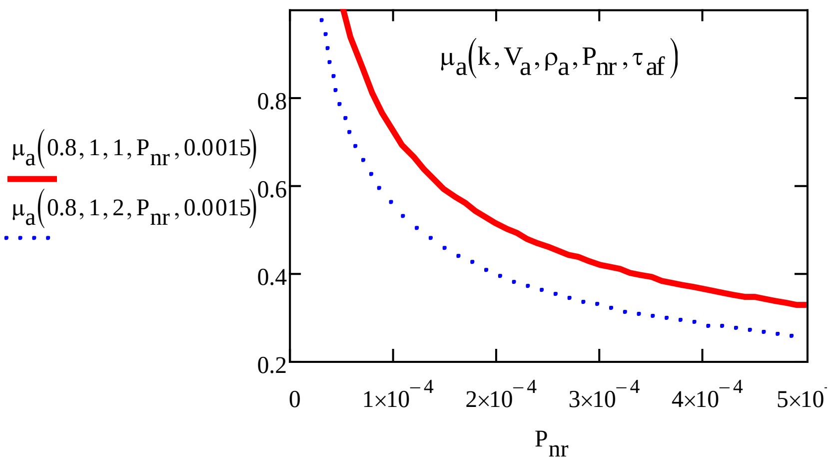


Fig. 7 The variation of the adhesion component on a steel ball with normal load

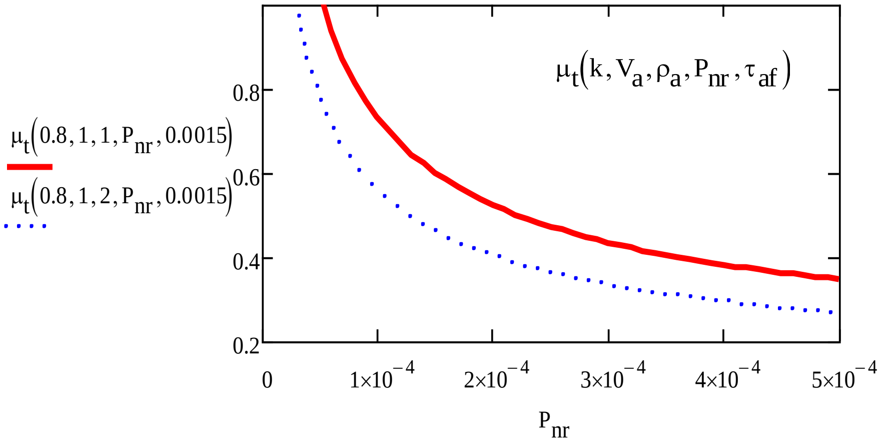


Fig. 8 Total coefficient of friction versus normal load

In Fig. 8 the deformation component of the friction coefficient (hysteresis component) is very small compared to the adhesion component. In this case, the effect of hysteresis component is very small.

3. Experimental method

The experimental set-up for measurement of rubber friction with steel ball is shown in Fig.9.

- The material of the band is rubber, which is used in coulombian damper for washing machine.
- The rubber band with a size of 30*12*4 mm was fixedly glued.
- The steel ball has a radius R = 10 mm and is much harder than the rubber band. It was mounted on a force sensor.

During the measurement, the steel ball was pressed into the rubber band with a given constant normal force which was applied in the device, and then the rubber band was moved horizontally with a constant velocity. The frictional force and normal force were measured by a tangential sensor and force cell sensor respectively. We experimentally investigated the friction coefficient of a steel ball sliding on a rubber base for different sliding velocities. The sliding velocities (2.5, 3.75, 5 mm/min) and the normal loads (4.5, 9 N) are used. Data for any parameter set (velocity, normal load) were averaged over 10 measurements.

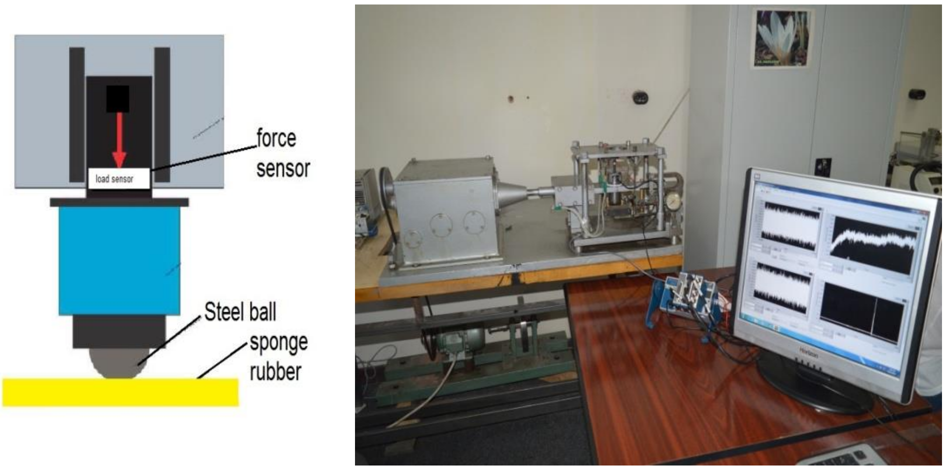


Fig. 9 Experimental set-up for measurement of coefficient of friction between a steel ball and a rubber

4. Results

The experimental results show friction parameters evolution. Fig. 10 and 11 represent experimental values obtained for the friction force at normal loads 4.5 N, 9 N and different sliding velocities 2.5, 3.75, 5mm/min.

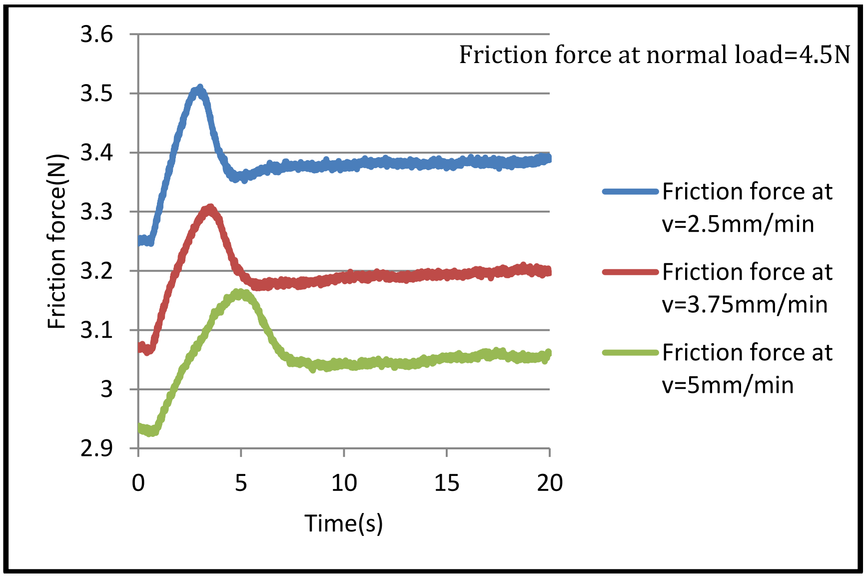


Fig. 10 Friction force vs. time at normal load 4.5N

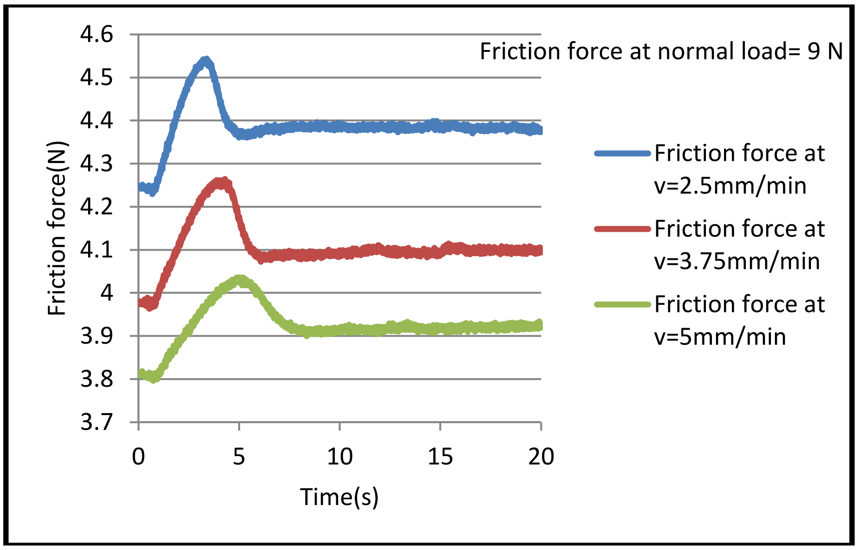


Fig. 11 Friction force vs. time at normal load 9N

Fig. 12 and 13 represent the experimental values for that the friction coefficient of viscoelastic material againest steel ball at normal load 4.5 N, 9 N and different sliding velocities. The coefficient of friction decreases when the sliding velocity increases. This variation can be a necessary condition to appear the stick-slip phenomena.

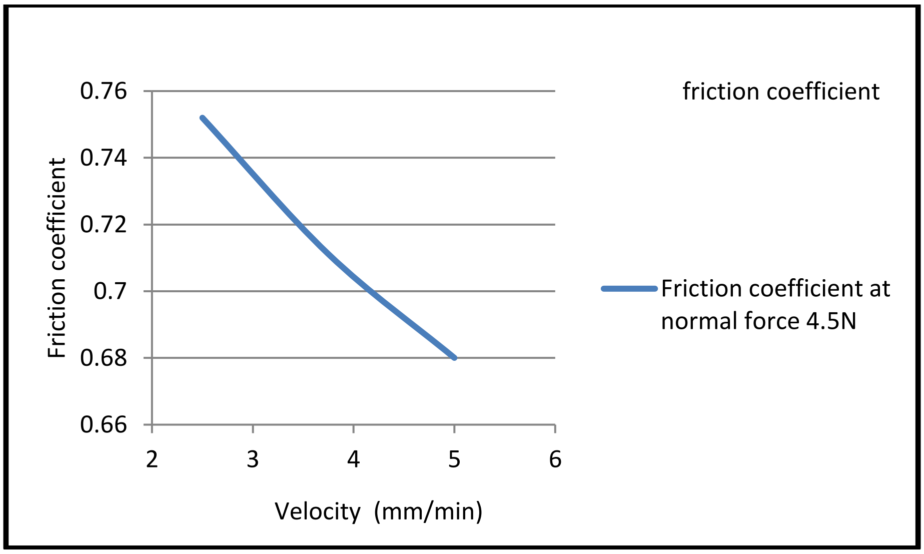


Fig. 12 Coefficient of friction vs. sliding velocity at normal load 4.5 N

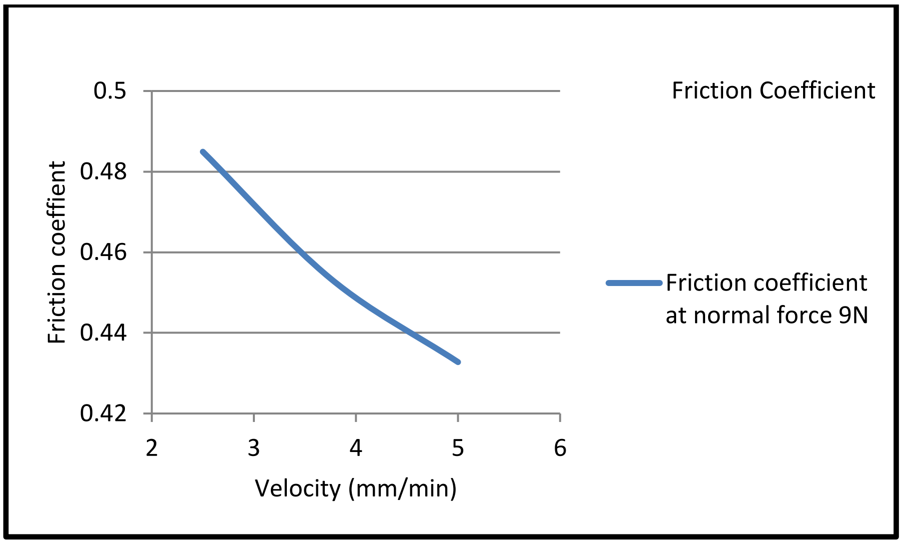


Fig. 13 Coefficient of friction vs. sliding velocity at normal load 9 N

Fig. 14 shows values of the total coefficient of friction with two penetration velocities and four experimental points (two velocities and two normal loads) which were mentioned in the experiments paragraph. This gives good matching between theoretical and experimental results.

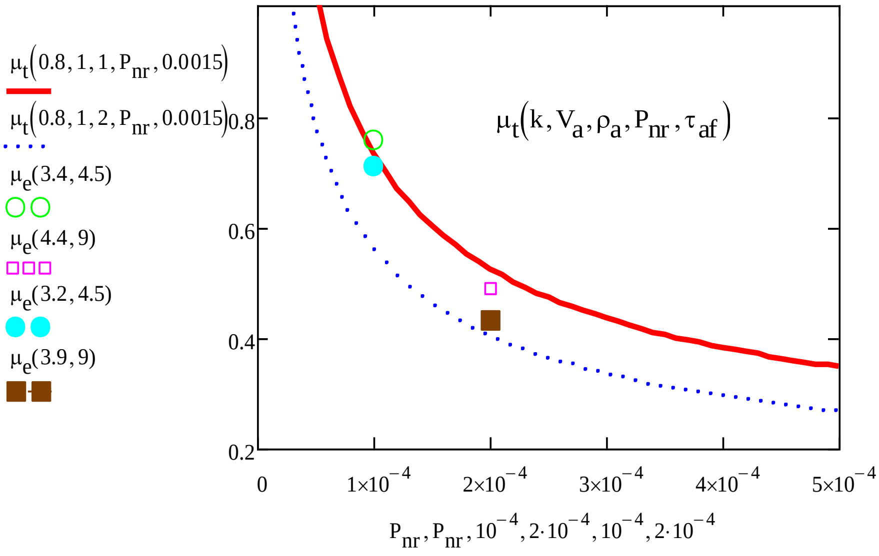


Fig. 14 Theoretical and experimental total coefficient of friction vs. normal load

5. **Conclusions**

In this paper a theoretical and experimental study concerning the coefficient of friction between rubber material and steel ball are presented. We analyzed the coefficient of friction of rubber (viscoelastic material) contacting with steel ball. The solutions shown the friction coefficient depends on sliding velocity and normal load. The Striebeck curve of the friction coefficient dependents on the relative sliding velocity is simulated as a parabola for the dry, limit and mixed friction regime, and being tangent to a line for the Newtonian fluid friction. The friction coefficient of hysteresis component increases when the normal load increases, but the friction coefficient of adhesion component decreases when the normal load increases. The friction coefficient of hysteresis component is very small compared to the friction coefficient of adhesion component. A good agreement between theoretical and experimental results is obtained.

R E F E R E N C E S

[1]. *N. Sclater., N. Chironis*, Mechanisms and mechanical devices sourcebook, McGraw-Hill, New York, 2001.

[2]. *L. Skedung, et al*. Finger Friction Measurements on Coated and Uncoated Printing Papers. Tribol. Lett. **vol.** 37, (2009), pp. 389–399.

[3]. *K. Holmberg, P. Andersson, A. Erdemir*, Global energy consumption due to friction in passenger cars. Tribol. Int. **vol.** 47, (2012), pp. 221–234.

[4]. *V.L. Popov*, Contact Mechanics and Friction, Physical Principles and Applications, Springer, Berlin, 2010.

[5]. *S. Chu, A. Majumdar*, Opportunities and challenges for a sustainable energy future. Nature **vol.** 488, (2012), pp. 294–303.

[6]. *O. Ben-David, J. Fineberg*, Static Friction Coefficient Is Not a Material Constant. Phys. Rev. Lett. **vol.**106, no. 254301 (2011).

[7]. *S.M. Rubinstein, G. Cohen, J. Fineberg*, Detachment fronts and the onset of dynamic friction. Nature **vol.** 430, (2004), pp. 1005–1009.

[8]. *A. Schallamach*, Friction and abrasion of rubber. Wear **vol.** 1, (1958), pp. 384–417.

[9]. *M. Barquins*, Sliding friction of rubber and Schallamach waves — A review. Mater Sci. Eng. **vol.** 73, (1985), pp. 45–63.

[10]. *Li Q., M. Popov, A. Dimaki, A.E. Filippov, S. Kürschner, V.L. Popov*, Friction between a viscoelastic body and a rigid surface with random self-affine roughness. Phys. Rev.Lett. **vol.** 111, no. 034301, (2013).

[11]. *H. Shahad, A.Tudor, M Vlase, N Cerbu, K Ahmad*, The effect of friction in the Columbian damper, Conference on Tribology, Galati 2016, Romania, IOP Conf series: Materials science and Engineering **vol.** 174, no. 012021(2017).

[12]. *Al Sayed Baraa, E. Chatelet, S. Baguet, J.K. Georges,* Dissipated Energy And Boundary Condition Effects Associated to Dry Friction on the Dynamics of Vibrating Structures, Mechanism and Machine Theory **vol**.46, (2011), pp. 479–491.

[13]. N.*N. Pashkov, N.A. Kil*, Engineering Mechanics for Constructors, Moscow: Vysshaya Shkola, 1969.
[14]. *I. A. Soldatenkov*, Calculation of the Deformation Component of the Force of Friction for a Standard Elastoviscous Base, Journal of Friction and Wear, **Vol. 29**, no. 1, (2008), pp. 7–14.