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RESIDUAL ELASTO-PLASTIC STRESSES ANALYSIS OF POLYMERIC THICK –WALLED PRESSURIZED CYLINDER

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Abstract

A theoretical solution is presented for polymeric thick pressurized cylinder, where material behaviour is described by the modified Von Mises criterion. The solution is carried out using different values of Y_c/Y_t ratio to demonstrate their effects on the plastic zone radius and on the radial and hoop stresses also on the residual stress components.

The results is indicated that the influence of α ratio or (Y_c/Y_t) ratio on the plastic zone radius and stress distributions is significant, and it can be shown that , when α ratio increases the plastic zone radius decreases , the value of α ratio has no effect on the value and distribution of radial stress, and when α ratio increases the level of the hoop stress increases in the plastic zone and decreases in the elastic zone and the value of α ratio is directly proportional to the value of residual radial and hoop stresses in the cylinder. A published finite element results give a reasonable agreement with the obtained results.

تحليل الاجهادات المرنة – اللدنة المتبقية للاسطوانه البوليميرية السميكة الجدران والمعرضة لضغوط داخلية

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الخلاصة

في هذا البحث تم دراسة وتقديم حل نظري للاسطوانات البوليميرية السميكة الجدران. لقد استخدم لوصف سلوك المادة معيارية فون ميسز المطورة. الحل تم باستعمال قيم مختلفة لـ Y_c/Y_t لتوضيح تأثيرها على نصف قطر مجال اللدونة وعلى توزيع الاجهادات القطرية والحلقية وكذلك مركبات الاجهادات المتبقية. تشير النتائج ان تأثير نسبة Y_c/Y_t او نسبة α واضحة على مجال نصف قطر اللدونة وعلى توزيع الاجهادات ، حيث تبين انه عند زيادة نسبة α فان نصف قطر مجال اللدونة سوف يقل وان قيمة α لا تأثير لها على قيم وتوزيع الاجهاد القطري، حيث انه عند زيادة α فان الاجهاد الحلقي سوف يزداد في مجال اللدونة ويقل ضمن مجال المرونة، حيث ان قيمة α تتناسب طرديا مع قيمة الاجهاد المتبقي والاجهاد الحلقي في الاسطوانة. تم مقارنة نتائج الحل النظري مع نتائج طريقة العناصر المحددة، والتي اظهرت تقاربا مقبولا.

Introduction

The formulation of elasto-plastic relation for a complex problem under multiaxial stresses can be achieved by assuming a reasonable mathematical model to correlate between the uniaxial test results and the multiaxial cases.

The general relation between stress and strain can be obtained in terms of the uniaxial behaviour, by specifying the following rules and conditions [1].

(i) The elastic stress–strain relations, (ii) An initial yield condition, (iii) A flow rule which relates the plastic strain increments to the stresses and stress increments, and (iv) A hardening rule for establishing the conditions for subsequent yield from a plastic state. It is necessary to have an initial yield condition which characterize the transition of a material from the elastic state to the state of yielding under any possible combination of stresses. One of the most widely used yield criteria for metallic materials is the Von Mises criterion. The Von Mises criterion is based on the assumption that the hydrostatic stress has no effect on yielding of metallic materials, i.e the only effective component is the deviatoric stress. While for polymetric materials, the following additional points have to be taken into consideration [2,3] (i)The hydrostatic stress effect, (ii) Tensile and compressive yield stresses are not necessarily equal.

The modified pressure Von Mises yield criterion takes into account the above two points. The actual variations between the Von Mises and the modified Von Mises yield criterion can be seen in the present work by comparing the results of the solution for the thick cylinder. Hani [4] studied the thermo-elasto- plastic behavior of the thick wall pressurized cylinder for metallic and polymeric materials by finite element method. Rogge et al [5] use the FEM package "NONSAP" to compare between the Von Mises and modified Von Mises yield criteria. Najdat et al [6] studied the effect of yield stress on the stress distribution of thick walled pressurized cylinder using finite element method and Davidson et al [7] studied the residual stresses in thick walled cylinders resulting from mechanically induced overstrain. Hani [8] studied the theoretical elasto-plastic analysis of thick pressurized cylinder using tresca yield criterion. In this study the complete theoretical elasto-plastic solution and calculating the residual stresses for thick walled cylinder is presented using modified Von Mises yield criterion.

Pressure Modified Von Mises Yield Criterion

The distortion energy theory or Von Mises criterion assumes that yielding begins when the distortion energy in a multiaxial problem is equal to the distortion energy at yielding in a simple uniaxial test. Using this criterion or condition, it can be shown that the yield surface in a three dimensional stress space is represented by [9]

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2=2Y^2$$

By substituting Y_c and Y_t in place of Y^2 in the above equation, it can be deduced that:

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2+C_1(\sigma_1+\sigma_2+\sigma_3)=2Y_cY_t \quad \text{.........................(1)}$$

Where: C_1 is a constant to be determined.

To find the value of C_1, the uniaxial case can be considered as follows:

$$\sigma_1=\sigma_x \;\; , \; \sigma_2=\sigma_3=0$$

Then equation (1) becomes:

$$2\sigma_x{}^2+C_1\sigma_x=2Y_tY_c$$

or

If σ_x equals to the yield stress (Y_t or Y_c), then it can be shown that:
If $\sigma_x = Y_t \quad \Rightarrow \quad C_1 = 2(Y_c - Y_t)$
If $\sigma_x = -Y_c \quad \Rightarrow \quad C_1 = 2(Y_c - Y_t)$
Substituting for C_1 , equation (1) becomes

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2+2(Y_c-Y_t)(\sigma_1+\sigma_2+\sigma_3)=2Y_cY_t \quad \text{...... ....(2)}$$

The above equation is similar to the equation of modified Von Mines mentioned in Ref.[2] and [3], and it can be rewritten as follows:

$$\bar{\sigma}_e = 2Y_e$$

Where

$$\bar{\sigma}_e = \sqrt{\bar{\sigma}^2 + 3\nabla\sigma_m}\ ,\ \sigma_m = \frac{1}{3}(\sigma_1+\sigma_2+\sigma_3)\ ,\ \bar{\sigma} = \sqrt{3}J_2\ ,$$

$$J_2 = \frac{1}{2}[(\sigma_1-\sigma_m)^2+(\sigma_2-\sigma_m)^2+(\sigma_3-\sigma_m)^2],\ \text{and}\ \nabla = Y_c - Y_t\ ,\ Y_e = \sqrt{Y_cY_t}$$

Elastic Expansion and Initial Yielding

Consider a thick-walled cylinder, with inner radius (a), and the external radius (b), which is subjected to an internal pressure “P” . The material is assumed to be elastic ideally plastic. The equilibrium equation for the pressurized cylinder can be written as follows [10,11]:

$$\frac{\partial\sigma_r}{\partial r} = \frac{\sigma_\theta - \sigma_r}{r} \quad \text{...(3)}$$

This equilibrium equation can be integrated to obtain the general elastic solution:

$$\sigma_r = A_v[1-\left(\frac{b}{a}\right)^2] \quad \text{and} \quad \sigma_\theta = A_v[1+\left(\frac{b}{a}\right)^2] \quad \text{..................................(4)}$$

Where: $A_v = \dfrac{P}{\lambda^2 - 1}$ and $\lambda = \dfrac{b}{a}$

This is known as Lame’s equation. The longitudinal stress for plane strain condition ($\varepsilon_z = 0$) may be written from Hooke’s law as:

$$\sigma_z = \upsilon(\sigma_r + \sigma_\theta) = 2\upsilon A_v$$

If the material yields according to the modified Von Mises criterion, i.e polymeric materials, it can then be shown from equation (2) that:

$$(\sigma_r-\sigma_\theta)^2+(\sigma_\theta-\sigma_z)^2+(\sigma_z-\sigma_r)^2+2(Y_c-Y_t)(\sigma_r+\sigma_\theta+\sigma_z)=2Y_cY_t$$

Which can be simplified to:

$$A_v^{\,2}\,[\,3\left(\frac{b}{r}\right)^4 + (1-2\upsilon)^2 + 2(Y_c-Y_t)A_v(1+\upsilon)] = Y_cY_t \quad \text{..................................(5)}$$

The second and third terms on the left-hand side are independent of r, and the term $(\frac{b}{r})^4$ has the greatest value where r = a . Hence yielding begins at the inner radius when the applied pressure becomes:

$$P = \frac{Y_t(\lambda^2-1)}{\sqrt{3(\frac{b}{r})^4 + (1-2\upsilon)^2}} \quad \text{..(6)}$$

Where $\alpha = \frac{Y_c}{Y_t}$, and P is the minimum value of the internal pressure to initiate yield in the cylinder

Elastic- Plastic Expansion

When the internal pressure exceeds P , a plastic zone spreads out from the inner radius, hence the elastic-plastic boundary at any stage becomes of radius ‘c’. The stress distribution in the elastic zone (i.e at $c \le r \le b$) can be written as follows:

$$\sigma_r = A'_v[1 - \left(\frac{b}{r}\right)^2] \qquad \text{.......(7)}$$

and $$\sigma_\theta = A'_v[1 + \left(\frac{b}{r}\right)^2] \qquad \text{.......(8)}$$

where A'_v can be obtained from equation (5) with r = c as follows:

$$A'_v = \frac{(1-\alpha)(1+\upsilon) + \sqrt{(\alpha-1)^2(1+\upsilon)^2 + \alpha[3(\frac{b}{c})^4 + (1-2\upsilon)^2]}}{3(\frac{b}{c})^4 + (1-2\upsilon)^2} Y_t \qquad \text{.......(9)}$$

The radial stress component in the plastic zone can be evaluated from the equilibrium equation by means of Runge-Kutta method. While the hoop stress component can be obtained for a given value of radial stress, provided that:
$\overline{\sigma}_\theta = 2Y_cY_t$, by means of Newton-Raphson algorithm .

Residual Stresses

The result of simple tensile test shown that when materials are loaded beyond the yield point the resulting deformation does not disappear completely when load is removed and the material is subjected to permanent deformation or so called permanent strain[7] . Residual stresses are therefore produced. In order to determine the magnitude of these residual stresses in the thick cylinder it is normally assumed that the unloading process from either partially plastic or fully plastic states is completely elastic. The unloading stress distribution is therefore linear and it can be subtracted graphically from the stress distribution in the plastic or partially plastic state to obtain the residual stresses.

Suppose that a thick-walled cylinder which is rendered partially plastic by the application of an internal pressure (P) is completely unloaded by releasing the pressure. For sufficiently small value of (P). Then the residual components can be written as follows:

For the region $(c \le r \le b)$

$(\sigma_r)_R = (\sigma_r)_1 - (\sigma_r)_e$, $(\sigma_h)_R = (\sigma_h)_1 - (\sigma_h)_e$

For the region $(a \le r \le c)$

$(\sigma_r)_R = (\sigma_r)_2 - (\sigma_r)_e$, $(\sigma_h)_R = (\sigma_h)_2 - (\sigma_h)_e$

Where

$(\sigma_r)_R$, $(\sigma_h)_R$ residual hoop and radial stresses respectively .

Case studies

Case one

In this case a thick cylinder is subjected to an internal pressure of (1000, 1050, 1100, 1150, 1200) psi which is equivalent to (6.895, 7.239, 7.584, 7.929, 8.274) MPa respectively. The modified Von Mises criterion is assumed with different values of (Y_c/Y_t) or of α ratio. The main dimensions of this case and material properties are as follows:

a = 1.333 in (3.385 cm), b= 2.0 in (5.08 cm), E=134000 psi (923.93 MPa), with elastic ideally plastic material υ=0.47, Y_t=2750 psi (18.961 MPa) , α = 1.0, 1.2, 1.35, 1.5, and 2.0 .

Fig(1) shows the location of the elastic-plastic interfaces as a function of α ratio and the internal pressure. The theoretical results show a reasonable agreement with the finite element results published by [5]. However, it is clear that the FEM results begin to diverge from the theoretical at high plastic deformation. The deviations between the two sets of results increase with the increasing of the pressure.

Fig(2) shows the variation of the plastic percentage of the cross-section with the internal pressures at different values of α ratio. It can be deduced that the plastic zone radius is increased with decreasing the value of α ratio. This example can be considered as a verification case study for the theoretical solution and its numerical algorithms.

Case Two

In this case, thick cylinder is tested with b/a ratio equals to 2. This cylinder is subjected to an internal pressure which is varied from 12 to 20 MPa. The main dimensions and material properties are as follows:

a=100 mm , b=200 mm, $\upsilon = 0.4$, Y_t=24MPa , α = 1.0, 1.25, and 1.5

Figs(3-5) show the distributions of the dimensionless radial stress (σ_r / Y_t) over traverse section of the cylinder at different values of α ratio and with internal pressure of 12, 14, and 16 MPa. The results indicate that the α ratio has no influence on the distribution of the radial stresses. While the α ratio has great influence on the value and distribution of the dimensionless hoop stress (σ_θ / Y_t) as shown in Figs(6-8).

It is clear from Figs(6-8), that when the internal pressure increases, the plastic zone spread out in the cylinder of α = 1 is more rapid than that of α = 1.25 and which is in turn more rapid than of α =1.5 . Fig(6), where the applied internal pressure equals to 12 MPa, shows that the yielding occurs in cases of α = 1 and of α =1.25 and therefore the behaviour of the cylinder material is elastic–partially plastic. While for case of α =1.5 there is nearly no yielding and the stress is almost but not quite elastic.

Figs(9-11) show the variation of the dimensionless residual radial stress over the traverse section of the cylinder. At a given value of the internal pressure, it can be deduced that there is an indirect proportionality between α ratio and the dimensionless residual radial stress. In Fig(9) the residual radial stress for the case of α =1.5 equals to positive value while it should be negative or nearly equals to zero. The positive value is due to the numerical instability occurring in the calculations as a result of the very small plastic deformation in the cylinder.

Also an indirect proportionality between α ratio and the dimensionless residual hoop stress can be seen from Figs(12-14). It can be seen that in the case where the internal pressure is 12 MPa (or P/Y_t= 0.5) with α =1.5 the residual hoop stress is nearly equal to zero as a result of the only elastic deformation in the cylinder**,** while for α less than

1.5 the amount of plastic deformation in the cylinder is increased and therefore the value of the residual hoop stress is also increased.

Fig(15) shows the variation of the dimensionless plastic zone radius (c/a) with the dimensionless internal pressure (P/Y_t). It is interesting to note that the plastic zone radius "c" is proportional to α ratio for a given internal pressure. It is clear that at P/Y_t=0.5 and for case of α=1.5 the plastic radius is nearly equal to the inner radius of the cylinder.

Conclusions

A new theoretical elastic ideally plastic model for polymeric thick-walled cylinder using the modified Von Mises yield criterion has been proposed. The theoretical results indicate that the influence of α ratio or (Y_c/Y_t) ratio on the plastic zone radius and stress distributions is significant, and it can be deduced that:

(i) When α ratio increases the plastic zone radius decreases.

(ii) The value of α ratio has no effect on the value and distribution of radial stress.

(iii) When α ratio increases the level of the hoop stress increases in the plastic zone and decreases in the elastic zone.

(iv) The value of α ratio is directly proportional to the value of residual radial and hoop stresses in the cylinder.

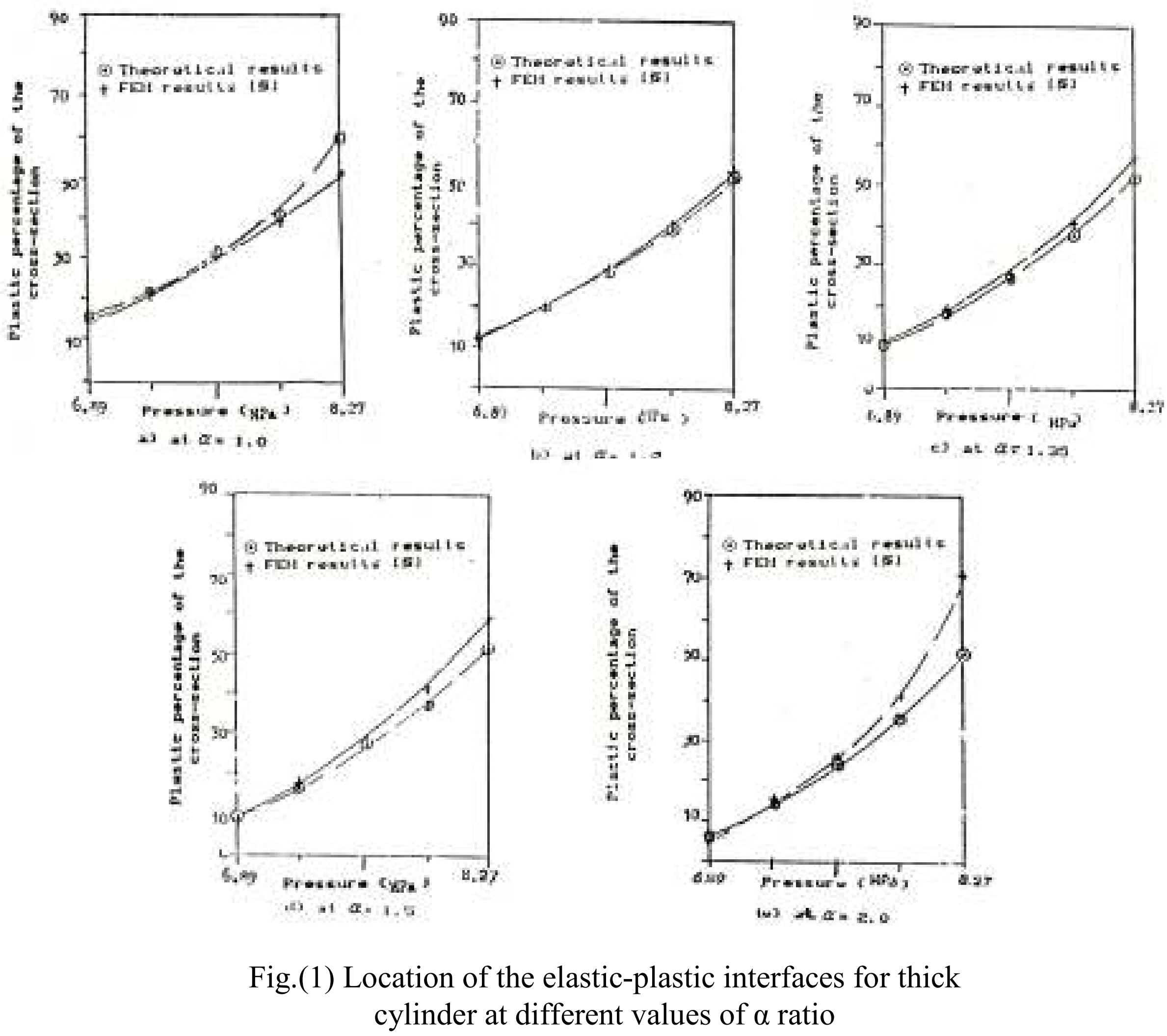

Fig.(1) Location of the elastic-plastic interfaces for thick cylinder at different values of α ratio

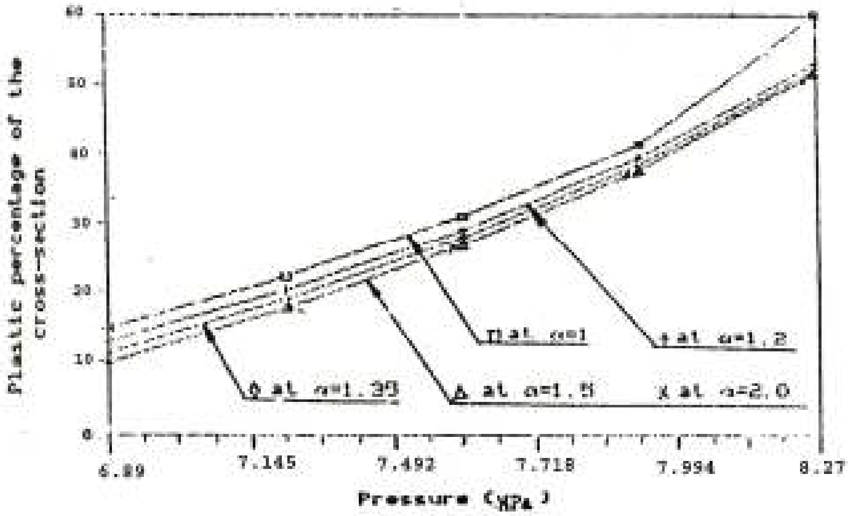

Fig.(2) Location of the elastic-plastic interfaces for thick cylinder at different values of α ratio (theoretical results)

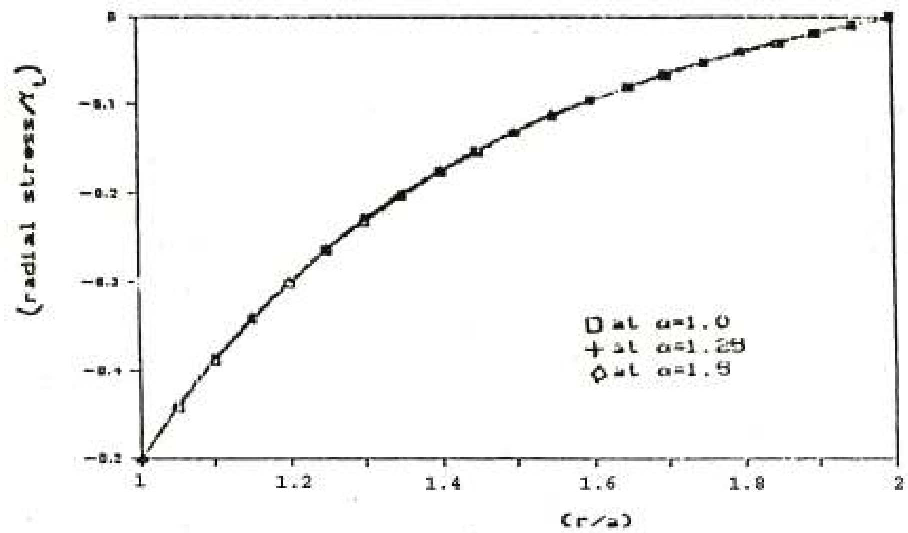

Fig.(3) Distribution of (σ_r / Y_t) with (r/a) at (P/Y_t) = 0.5

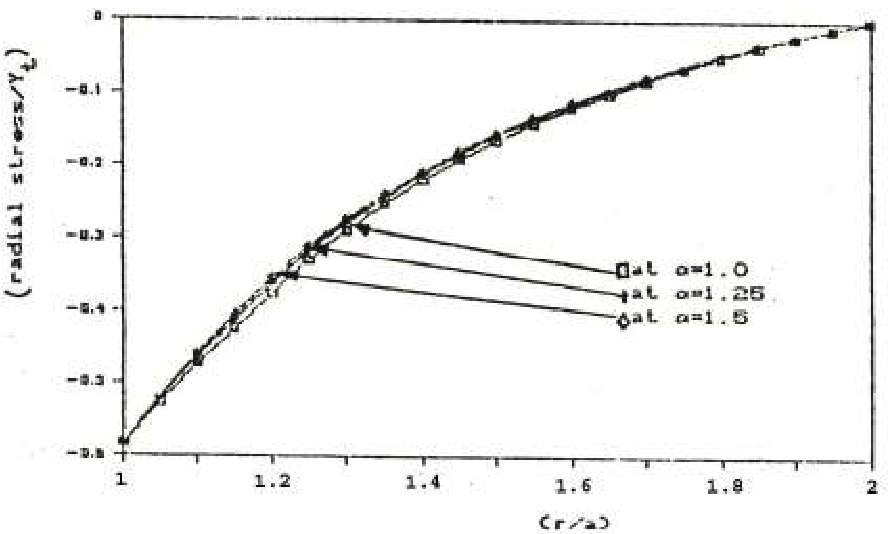

Fig.(4) Distribution of (σ_r / Y_t) with (r/a) at (P/Y_t) = 0.5833

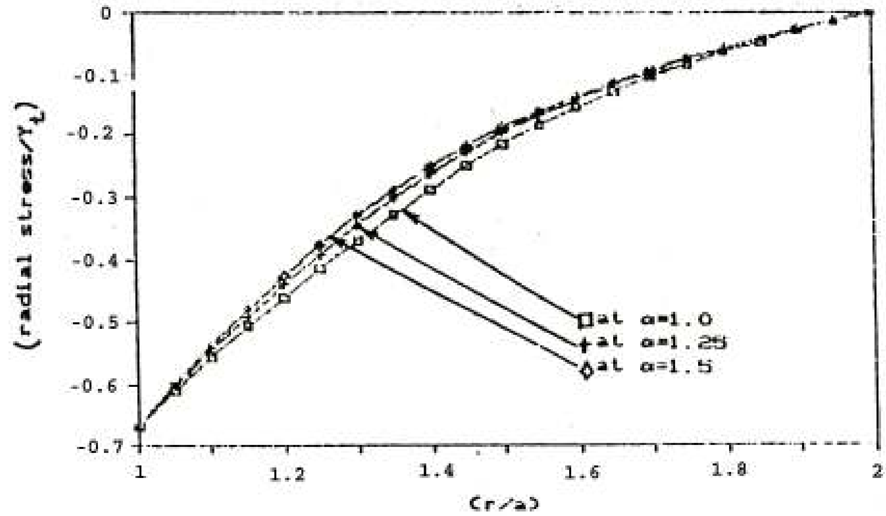

Fig.(5) Distribution of (σ_r / Y_t) with (r/a) at (P/Y_t) = 0.6

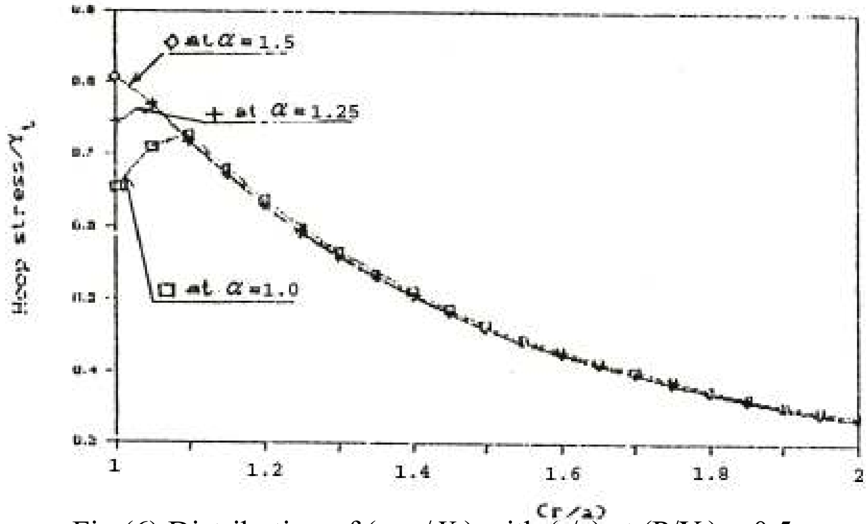

Fig.(6) Distribution of (σ_θ / Y_t) with (r/a) at (P/Y_t) = 0.5

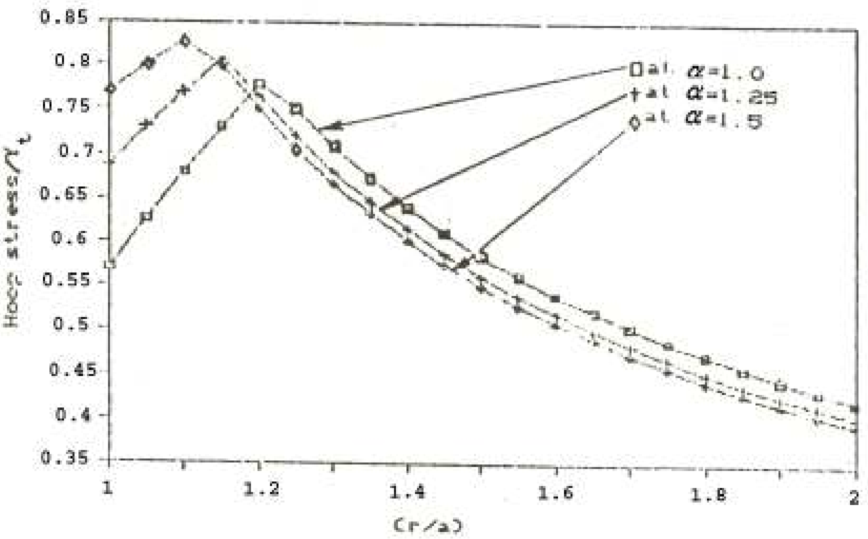

Fig.(7) Distribution of (σ_θ / Y_t) with (r/a) at (P/Y_t) = 0.5833

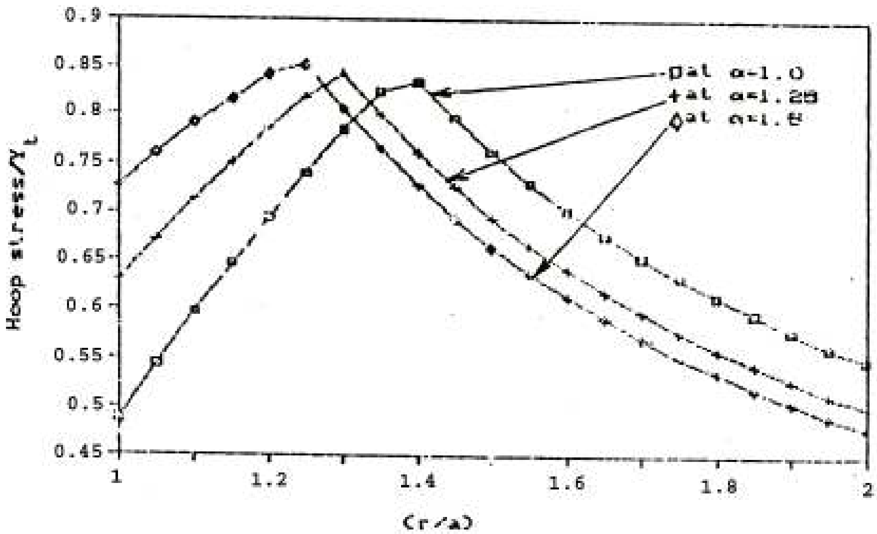

Fig.(8) Distribution of (σ_θ / Y_t) with (r/a) at (P/Y_t) = 0.6

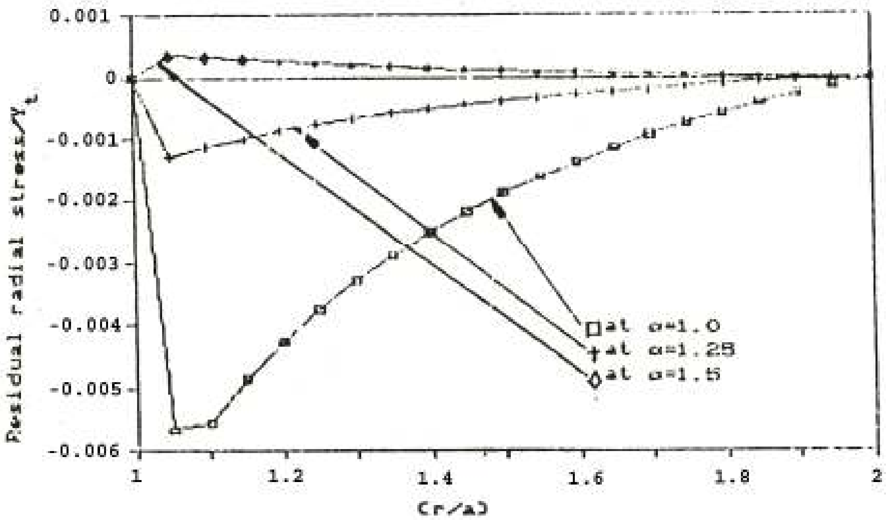

Fig.(9) Distribution of residual (σ_r / Y_t) with (r/a) at (P/Y_t) = 0.5

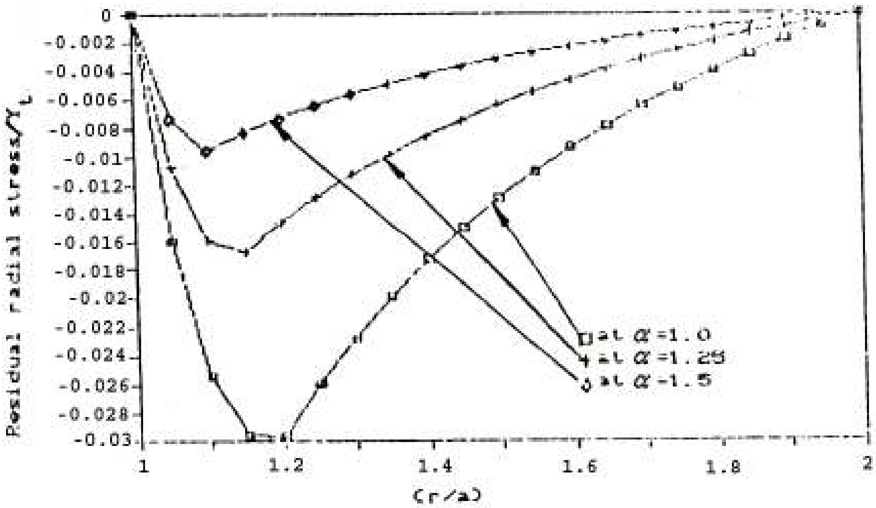

Fig.(10) Distribution of residual (σ_r / Y_t) with (r/a) at (P/Y_t) = 0.5833

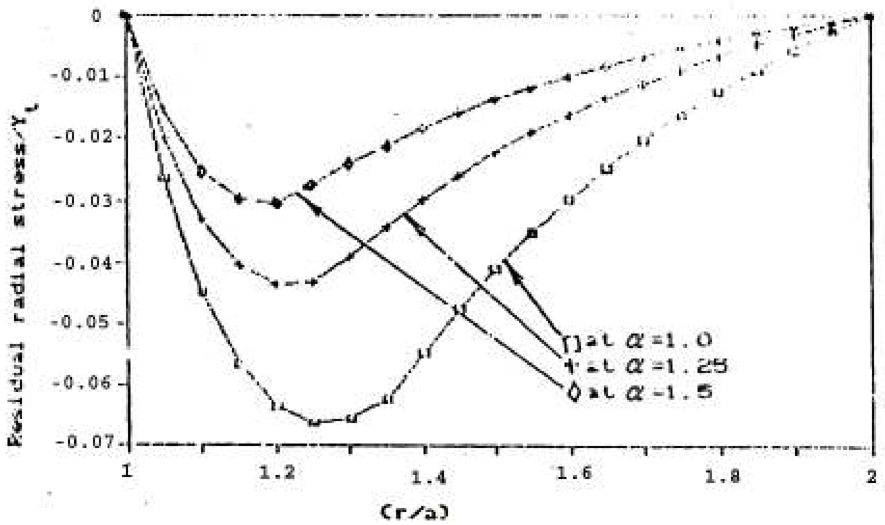

Fig.(11) Distribution of residual (σ_r / Y_t) with (r/a) at (P/Y_t) = 0.6

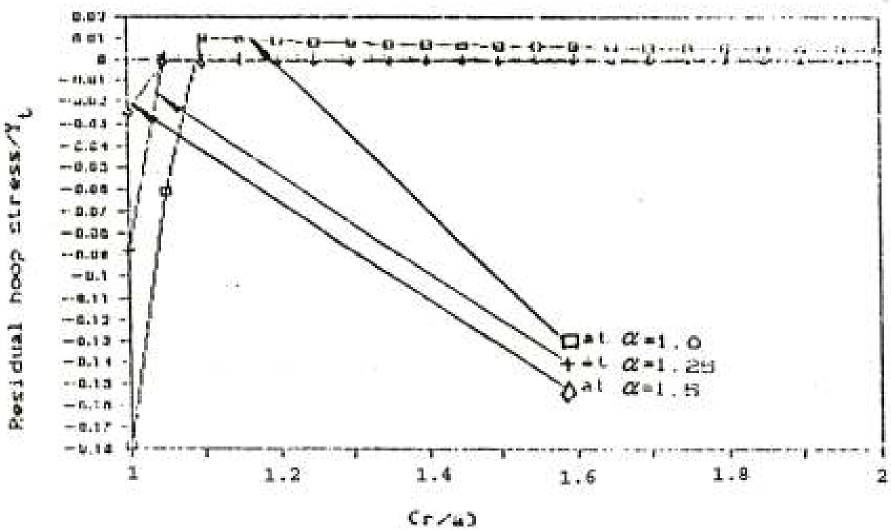

Fig.(12) Distribution of residual (σ_θ / Y_t) with (r/a) at (P/Y_t) = 0.5

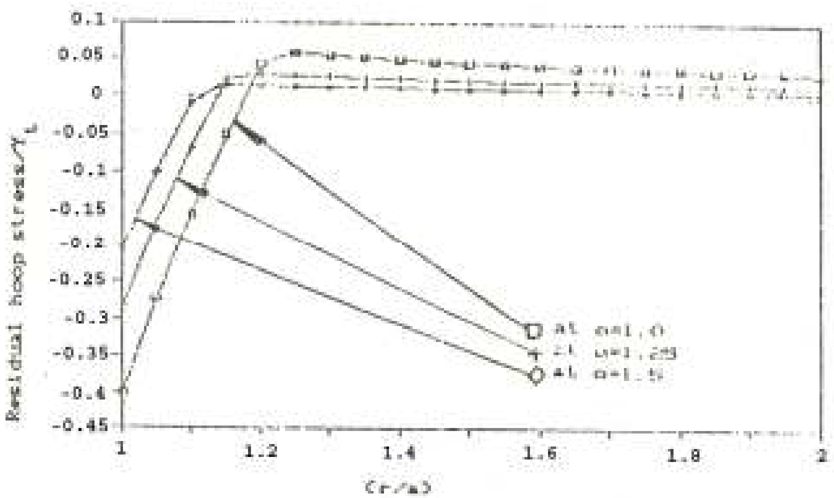

Fig.(13) Distribution of residual (σ_θ / Y_t) with (r/a) at (P/Y_t) = 0.5833

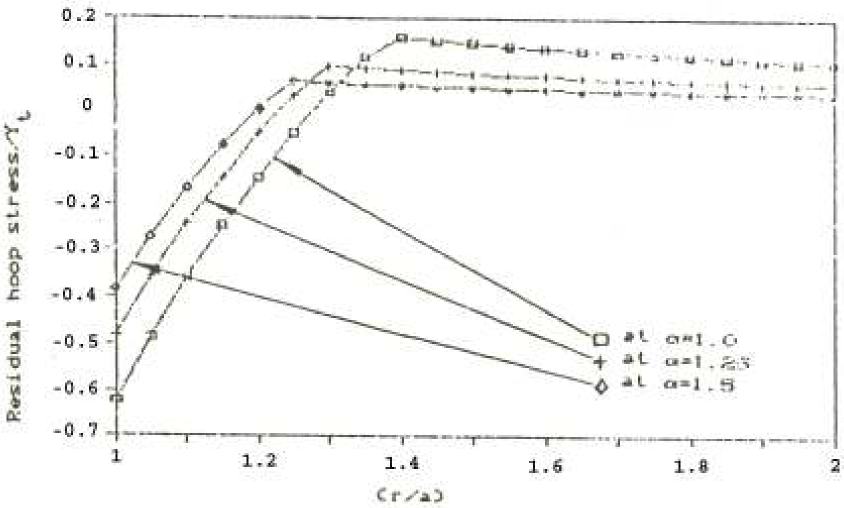

Fig.(14) Distribution of residual (σ_θ / Y_t) with (r/a) at (P/Y_t) = 0.6

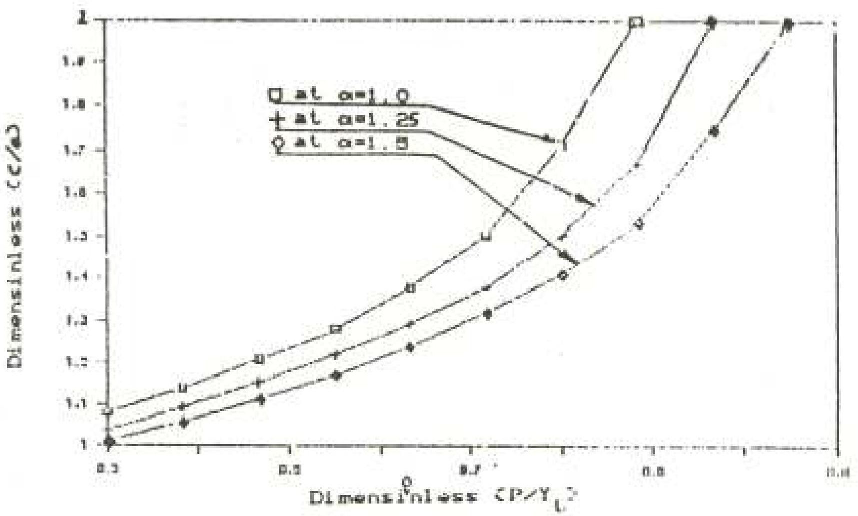

Fig.(15) Variation of (Plastic zone radius / a) with (P/Y_t)

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Notation

Symbol	Definition
a , b	inner and outer radius of cylinder
E	modulus of elasticity
α	ratio of Y_c/Y_t
P	internal pressure
c	elasto-plastic boundary radius
r	variable radius
$\sigma_1, \sigma_2, \sigma_3$	principal stress components
Y_t, Y_c	absolute tensile and compressive yield stresses, respectively
Y	yield stress (when Y_c=Y_t)
$\bar{\sigma}$	effective stress
$\bar{\sigma}_e$	equivalent effective stress
Y_e	equivalent yield stress
σ_m	mean or hydrostatic stress
J_2	second invariant of deviatoric stresses
υ	Possion's ratio
$\sigma_r, \sigma_\theta, \sigma_z$	radial, hoop , and longitudinal stress , respectively
$(\sigma_r)_e, (\sigma_h)_e$	elastic radial & hoop stress
$(\sigma_r)_1, (\sigma_h)_1$	plastic radial & hoop at $a \le r \le c$
$(\sigma_r)_2, (\sigma_h)_2$	plastic radial & hoop at $c \le r \le b$
$(\sigma_r)_R, (\sigma_h)_R$	residual radial & hoop stress
λ	ratio of $\frac{b}{a}$