INFLUENCE OF THE PROFILED ROTOR DESIGN ON THE PERFORMANCE OF ROTATING MACHINES

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Abstract. The aim of this paper is to expose the relation between the constructive form of the profiled rotor and the following parameters: the flow rate of the transported fluid, the driving power of the machine, the volumetric efficiency. Three constructive solutions of profiled rotors are presented. The flow rate of the fluid and the driving power of the machine are calculated for each one of them. The advantages and disadvantages of every constructive solution are outlined.
Keywords: rotating machines with profiled rotors.

1. INTRODUCTION

The term „rotating machine“ refers to the fact that the rotating machine presented in this paper can be used as:

– ventilator for the transport of gas and steam, pure or with suspensions;

– pump for the transport of the liquids, residual waters or every high viscosity fluid;

– low pressure compressor (blower), the gas having a discharge pressure of 1.5÷2.5 bar.

This type of rotating machine overtakes in performance the classical machines that have pistons with alternative rectilinear movement, because during a whole rotation the motor torque received from the shaft is almost integrally ceded to the transported fluid [1, 2] (fig.1):

$$M = F \cdot b \cdot \sin\alpha \qquad [\mathrm{N \cdot m}]$$

where: F – force that presses on the rotating piston; b – the force arm, namely the radius that traverses the ax of the piston; α – angle between arm and force, always equal to 90°.

Dynamically equilibrated constructive solutions of rotating machines were achieved by eliminating the slider-crank mechanism. Inertia and friction forces are lower than in the case of machines with pistons. Three constructive solutions are analyzed in the following:

– I[st] version: Profiled rotor with two blade-shaped pistons (§2.1);

– I[nd] version: Profiled rotor with two pistons of triangular shape (§2.2);

– III[rd] version: Profiled rotor with two pistons of curvilinear shape (§2.3).

2. ROTATING MACHINE FOR THE FLUID TRANSPORT

2.1. Rotating machine with profiled rotors in the shape of rectangular blades

The rotating machine is composed by a mobile subassembly (rotors, shafts) and a fixed subassembly (casings, intake and discharge branch pipes) (Fig. 1).

The rotors (2) and (5) rotate with the same speed inside the two casings (1) and (4); their synchronous rotation is insured by the use of a gearing consisting of two spur gears with the same pitch diameter, mounted on the shafts 7 and 9, outside the machine. The aspirated gas (fig.1.a) is transported towards discharge and after a 90° rotation of both rotors, positions from figure 1.b and subsequently, 1.c are reached. We do not consider the volume of the gas from the cavity, because it is brought back at intake when the contact between the piston and the upper rotor takes place. After a 180° rotation, the fluid from the useful volume Vu (fig 1.c), namely the space encompassed by the pistons, the lower casing (1) and the lower rotor (2), will be sent to the discharge chamber. After a complete rotation of the shaft (9) two such volumes will be transported from intake towards discharge [3] :

$$\dot{V}_u = 2\left(\frac{\pi R_c^2}{2} - \frac{\pi R_r^2}{2}\right) \cdot l \quad \left[\mathrm{m^3/rot}\right] \qquad (1)$$

The radius of the casing *(R_c)* is the sum between the radius of the rotor *(R_r)* and the height of the piston (z):

$$R_c = R_r + z \quad [\mathrm{m}] \qquad (2)$$

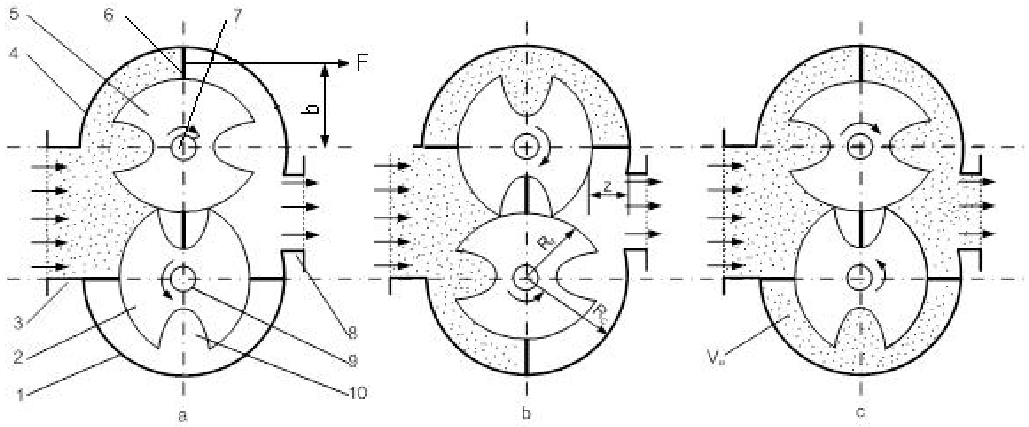

Fig. 1. Rotors with blade-shaped rotating pistons:
1 – lower casing; *2* – lower rotor; *3* – intake chamber; *4* – upper casing; *5* – upper rotor; *6* – blade – shaped rotating piston; *7* – driven shaft; *8* – discharge chamber; *9* – driving shaft; *10* – cavity where the upper rotor piston enters.

It results :

$$\dot{V}_u = \pi l z(z + 2\,R_r)\ [m^3/rot] \quad (3)$$

The volumetric flow rate discharged by a sole rotor of length *l* [m] and rotation speed *n* [rpm] will be :

$$\dot{V}_u = \pi l z\left(z + 2R_r\right)\cdot\frac{n}{60}\quad [m^3/s] \quad (4)$$

Because the machine has two identical rotors the flow rate transported by the machine will be:

$$\dot{V}_m = 2\dot{V}_u = \pi l z\left(z + 2R_r\right)\cdot\frac{n}{30}\quad [m^3/s] \quad (5)$$

For the given values: l=0.05 m; z=0.03 m; R_r=0.05 m; n_r= 300 rpm.
It results :

$$\dot{V}_m = \pi\cdot 0.05\cdot 0.03\left(0.03 + 2\cdot 0.05\right)\cdot\frac{300}{30} \quad (6)$$

$$\dot{V}_m = 0.006123\ \left[m^3/s\right] = 22.042\ m^3/h \quad (7)$$

Obviously, it is difficult to attach the blade to the rotor for the I[st] version; the case of a theoretical compressor was analyzed. Relations for computing the flow rate of the gas transported by the machine were established, in the hypothesis that the blade thickness was neglectable. For the versions II and III, the flow rate transported by the machine will be smaller because the volume occupied by the body of the triangular, respectively curvilinear piston is subtracted from V_u [4].

2.2. Compressors with profiled rotors having rotating pistons of triangular shape

The functioning of this new compressor type is similar with the one presented in paragraph 2.1 (Fig. 2). In this case, the blade is replaced by a triangle dimensioned observing materials strength constraints. The basis of the triangle was dimensioned, namely the section of the piston close to the rotor.

It is considered that a uniformly distributed charge p = 3 bar = $3\cdot10^5$ N/m^2 acts on the piston. A portion of the piston of length b = 1 cm and of height z = 3 cm is chosen; the value of h must be calculated (Fig. 3).

The concentrated force will be:

$$P = p\cdot A = 3\cdot10^5\cdot3\cdot10^{-4} = 90\ N \quad (8)$$

The bending moment:

$$M = P\cdot\frac{z}{2} = 90\cdot1.5\cdot10^{-2} = 1.35\,Nm \quad (9)$$

Elastic section modulus:

$$W_{nec} = \frac{M_{max}}{\sigma_{a\hat{i}}} \quad (10)$$

The rotors are made from cast aluminum, for which $\sigma_{a\hat{i}}$ is of 5÷7 N/mm^2 [16].

$$W_{nec} = \frac{1.35}{5\cdot10^6} = 0.27\cdot10^{-6}\ m^3 \quad (11)$$

For a rectangular section:

$$W_{nec} = \frac{bh^2}{6} = 0.27\cdot10^{-6}\ \left[m^3\right] \quad (12)$$

Because $b = 1\cdot10^{-2}$ m, it results the value of h:

$$h = \sqrt{6\cdot0.27\cdot10^{-4}} = 1.62\cdot10^{-2}\,m = 0.0162\ m = 1.62\ cm \quad (13)$$

Choosing the safety factor c = 1.85, it results h = 3 cm.

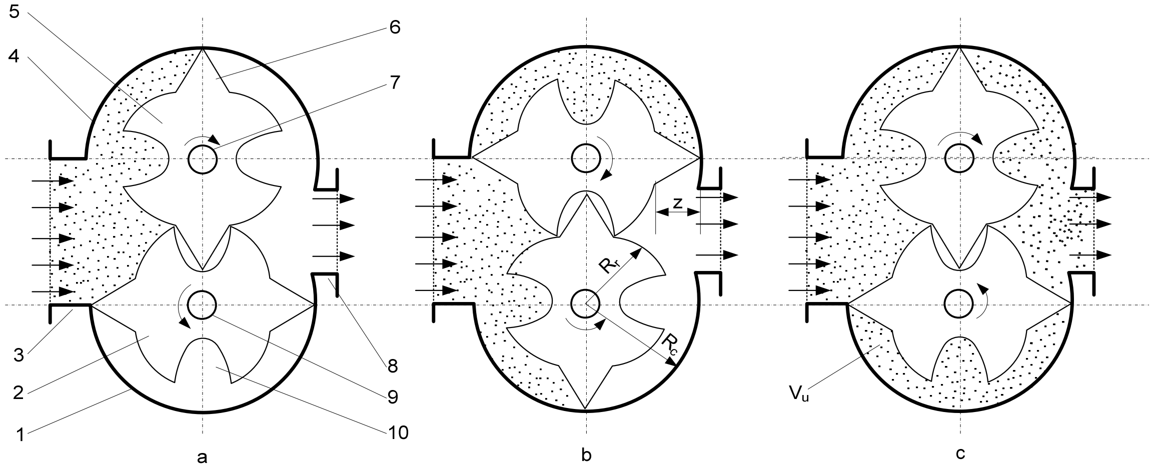

Fig. 2. Rotating volumetric compressor with triangular rotating pistons:
1 – lower casing; *2* – lower rotor; *3* – intake chamber; *4* – upper casing; *5* – upper rotor; *6* – rotating piston of triangular shape; *7* – driven shaft; *8* – discharge chamber; *9* – driving shaft; *10* – cavity where the upper rotor piston enters.

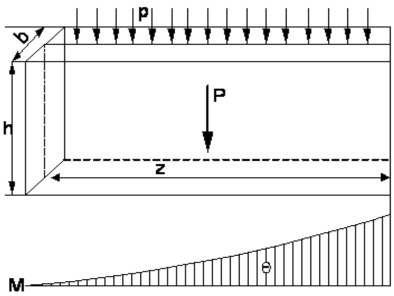

Fig. 3. Calculus notations.

This constructive solution guarantees a good strength of the piston and in the same time it creates two sealing zones: between the top of the piston and the inside of the casing and between the top of the piston and the cavity. As we can remark in Figure 4, the useful volume V_u is reduced with the volumes of the prisms *ABC* and *A'B'C'*; these are equal and together they form the volume of a piston of triangular section, namely a prism with the following dimensions: height: z = 30 mm; basis: b = 30 mm; length: l = 50 mm;

The area of the section between the basis of the prism and the rotor is neglected. The volume of this prism will be:

$$V_{p,II} = A_{bazei} \cdot l = \frac{1}{2} \cdot b \cdot z \cdot l = \frac{1}{2} \cdot 0.03 \cdot 0.03 \cdot 0.05 = = 0.0225 \cdot 10^{-3} \quad [\mathrm{m^3/rot}] \tag{14}$$

Compared with the theoretical flow rate transported by the machine in the version I:

$$\dot{V}_{m,I} = \pi lz(z + 2R_r) \cdot \frac{n}{30} \quad [\mathrm{m^3/s}] \tag{15}$$

the theoretical flow rate of the machine in this version II will be reduced with V_{pII}.

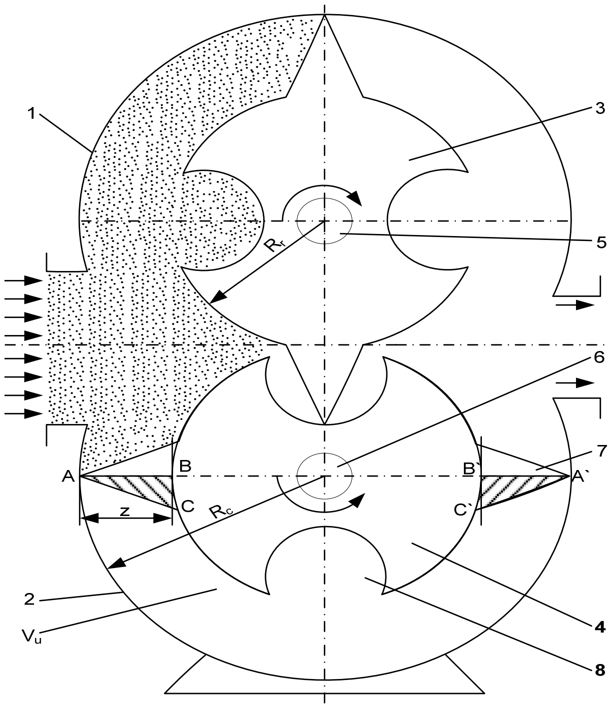

Fig. 3.6. Section through the rotating working machine:
1 – upper casing; *2* – lower casing; *3* – upper rotor; *4* – lower rotor; *5*, *6* – shafts; *7* – triangular piston; *8* – cavity.

The gas flow rate transported by a rotor:

$$\dot{V}_u = \left[\pi lz(z + 2R_R) - V_{p,II}\right] \quad [\mathrm{m^3/rot}] \tag{16}$$

The machine has two identical rotors, so that the flow rate transported will be:

$$\dot{V}_{m,II} = 2 \cdot \left[\pi lz(z + 2R_r) - \frac{1}{2}bzl\right] \cdot \frac{n}{60} \quad [\mathrm{m^3/s}] \tag{17}$$

For the same data as in version I and adding $b = 0.03$ m, we obtain a flow rate of:

$$\dot{V}_{m,II} = \left[\pi \cdot 0.05 \cdot 0.03\left(0.03 + 2 \cdot 0.05\right) - \right. \left. - \frac{1}{2} \cdot 0.03 \cdot 0.03 \cdot 0.05\right] \cdot \frac{300}{30} \quad (18)$$

$$\dot{V}_{m,II} = 0.0061219\,\mathrm{m^3/s} = 22.038\,\mathrm{m^3/h} \quad (19)$$

There is a flow rate difference between the versions I and II:

$$\dot{V}_{m,I} - \dot{V}_{m,II} = 22.042 - 22.038 = 0.00301\,\mathrm{m^3/h} \quad (20)$$

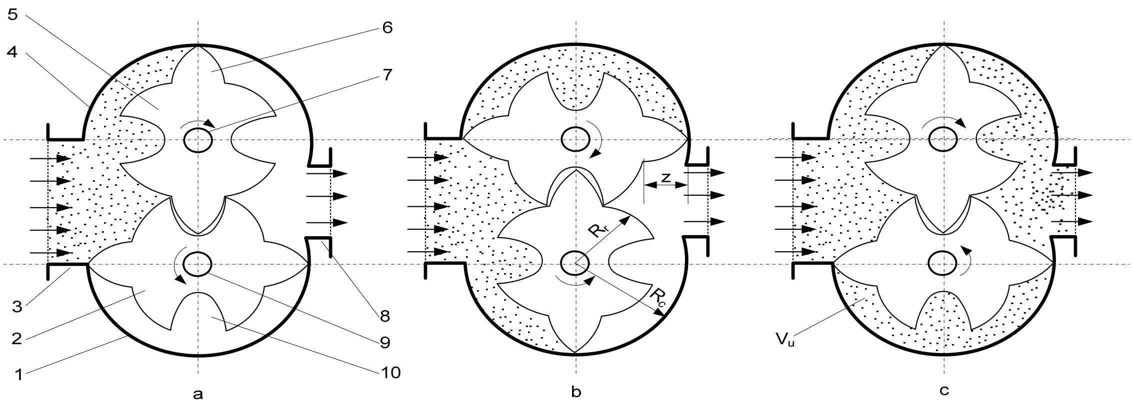

Fig 5. Transversal section through a rotating machine
1-lower casing; 2-lower rotor; 3-intake chamber; 4-upper casing; 5-upper rotor; 6-rotating piston of curvilinear shape; 7-driven shaft; 8-discharge chamber; 9-driving shaft; 10-cavity where the upper rotor piston enters.

2.3. Compressor with profiled rotors with curvilinear rotating pistons

In this constructive solution (variant III) the rotating pistons have a curvilinear shape (Fig. 5). This version was constructed in the laboratories of the POLITEHNICA University of Bucharest on the basis of results delivered by a computing software [5]. The working principle is the same as of versions I and II.

It can be remarked from Figure 6 that the useful volume V_u will be reduced with a volume equal to the areas (*ABC+A'B'C'*) multiplied with the length of the piston (l), dimension perpendicular on the plane of the figure. The areas ABC and A'B'C' are equal, hence the volume of a curvilinear piston ($V_{p,III}$) must be subtracted from V_u. This volume equals the piston area (S_p) multiplied by its length (l).

$$V_{p,III} = S_p \cdot l = 2 \cdot S_{ABC} \cdot l \quad [\mathrm{m^3}] \quad (21)$$

Hence the theoretical machine flow rate will be reduced with V_{pIII} leading to:

$$\dot{V}_u = \left[\pi l z\left(z + 2R_r\right) - \dot{V}_{p,III}\right] \; [\mathrm{m^3/rot}] \quad (22)$$

The flow rate transported by a rotor:

$$\dot{V}_u = \left[\pi l z\left(z + 2R_r\right) - \dot{V}_{p,III}\right] \cdot \frac{n}{60} \quad [\mathrm{m^3/s}] \quad (23)$$

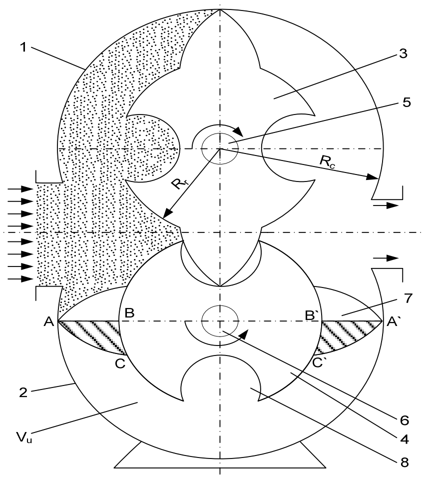

Fig. 6. Transversal section through the rotating working machine:
1 – upper casing; *2* – lower casing; *3* – upper rotor; *4* – lower rotor; *5,6* – shafts; *7* – curvilinear piston; *8* – cavity.

Because the machine has two identical rotors, the volumetric flow rate of the machine will be:

$$\dot{V}_{m,III} = 2\dot{V}_u = \left[\pi l z\left(z + 2R_r\right) - 2S_{ABC} \cdot l\right] \cdot \frac{n}{30} \; [\mathrm{m^3/s}] \quad (24)$$

The area of the surface S_{ABC} from Figure 6 is approximated with the area A_{ANM} from Figure 7, encompassed between the tangent to the rotor in the point N, namely the line MN, and the profile of a ½ of the piston, namely the curve AM.

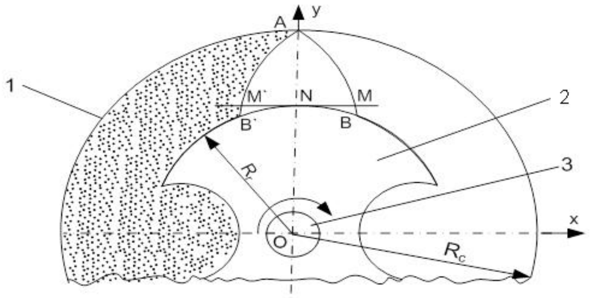

Fig. 7. Section through a rotor with a curvilinear piston: *1* – upper casing; *2* – upper rotor; *3* – shaft.

The area of the piston section will be:

$$S_p = 2S_{ABC} = 2S_{ANM} \;\; [m^2] \tag{25}$$

The coordinates of the points AN and M are $[x_i;y_i]$: $A(0;0.008)$; $N(0;0.05)$ and $M(0.01929; 0.04613)$ [5]. We divide the interval $NM \approx 0.02$ m in ten equal intervals of length 0.002 m. Hence the coordinates x_i of the points on OX are:

0.002; 0.004; 0.006; 0.008; 0.010; 0.012; 0.014; 0.016; 0.018; 0.020 m.

Corresponding to these values (x_i), the closest values x_i are chosen from the table containing the coordinates of the points. The values y_i are correspondingly taken from the table. The radius of the rotor (R_r = 0.05 m) is subtracted from values y_i. Values y_i* that decrease from y_i* = 0.03 m (point A) till y_i* = 0.00022 (point M) are obtained.

Table 1. **Coordinates of the points situated on the arc *AM***

No.	*x* [m]	Equivalent point no.	x_i [m]	y_i [m]	$y_i^* = y_i - 0{,}05$ [m]
0≡*A*	0	-	0	0.00800	0.03000
1	0.002	4	0.01720	0.07876	0.02876
2	0.004	8	0.00406	0.07696	0.02696
3	0.006	12	0.00621	0.07511	0.02511
4	0.008	16	0.00818	0.07323	0.02323
5	0.010	20	0.00998	0.07132	0.02132
6	0.012	25	0.01197	0.06894	0.01894
7	0.014	31	0.01402	0.06611	0.01611
8	0.016	38	0.01595	0.06289	0.01289
9	0.018	49	0.01807	0.05818	0.00818
10	0.020	72	0.01974	0.05022	0.00022

Numerical integration, properly the trapeze formula, was used for the calculus of the area of the curvilinear piston profile [6].

The first surface is equal to the basis (h) multiplied by the medium height of the trapeze, namely:

$$KL = \frac{f(x_o) + f(x_o + h)}{2} \tag{26}$$

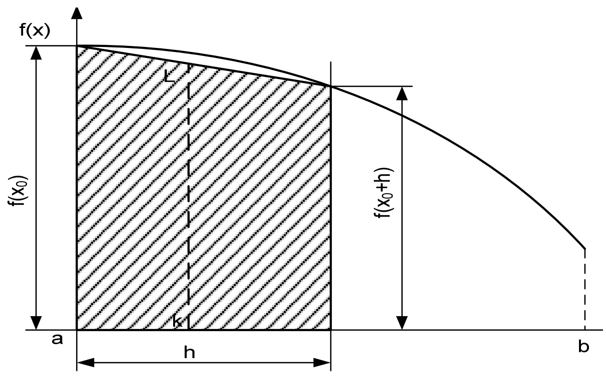

Fig. 8. Calculus notations.

The surface under the function graphics will be:

$$I = \int_a^b f(x)\mathrm{d}x \approx \sum_{i=0}^{N-1} \int_{x_i}^{x_{i+1}} f(x)\mathrm{d}x \tag{27}$$

$$I_1 = \frac{h}{2} \cdot \left(y_o + 2y_1 + 2y_2 + ...2y_{N-1} + y_N\right) \tag{28}$$

Using the values of table 1 we obtain:

$$\begin{aligned} I_1 &= \frac{0.002}{2}(0.0300 + 2\cdot 0.02876 + 2\cdot 0.02696 + \\ &+2\cdot 0.02511 + 2\cdot 0.02323 + 2\cdot 0.02132 + \\ &2\cdot 0.01894 + 2\cdot 0.01611 + 2\cdot 0.01289 + \\ &+2\cdot 0.0818 + 0.00022) = 3.937\cdot 10^{-4}\ \mathrm{m}^2 = \\ &= 3.937\ \mathrm{cm}^2 \end{aligned} \tag{29}$$

Hence: $S_{ABC} = S_{ANM} = 3.937{\cdot}10^{-4}\,m^2$ (30)

Replacing in formula (24), it results:

$$\begin{aligned} \dot{V}_{m,III} &= [\pi\cdot 0.05\cdot 0.03\cdot\left(0.03 + 2\cdot 0.05\right) - \\ &-2\cdot 3.937\cdot 10^{-4}\cdot 0.05]\cdot\frac{300}{30} \end{aligned} \tag{31}$$

$$\dot{V}_{m,III} = 0.005725 \quad m^3/s = 20.613 \quad m^3/h \tag{32}$$

In conclusion, the flow rate transported by the three versions of machines will be:

$$\dot{V}_{M,I} > \dot{V}_{M,II} > \dot{V}_{M,III} \tag{33}$$

$$22.042 > 22.038 > 20.613 \;\; [m^3/h] \tag{34}$$

or: $0.006123 > 0.006121 > 0.005725 \;\; [m^3/s]$ (35)

It is obvious that the volumetric efficiency (η_v) of the machine decreases when the flow rate transported by the machine decreases: $\eta_{v,I} > \eta_{v,II} > \eta_{v,III}$ [7].

The driving power of the rotating machine will be:

$$P = \dot{V}\,\Delta p \ \left[\mathrm{W}\right] \tag{36}$$

where Δp is the pressure increase between intake and discharge; obviously:

$$P_{m,I} > P_{m,II} > P_{m,III} \tag{37}$$

In the I[st] version, the machine transports a flow rate greater than in versions II and III but it needs also a greater driving power. The design of the rotors provides special sealing conditions, respectively.

The curvilinear shape of the piston provides a better contact between the rotating piston and the cavity of the adjacent rotor (Fig. 9).

From the figure 9 we remark the following:

– If the piston has a triangular shape, there is a sole contact line (zone A) of length l (perpendicular dimension on the plane of the figure) between the piston and the cavity of the adjacent rotor.

– If the piston has a curvilinear shape, from Figure 9 we remark that there are three contact lines: B, A and C; hereupon the fluid losses between intake and discharge will be reduced, diminishing the "reverse" flow, hence the volumetric efficiency will increase when the machine works as a pump. Consequently the effective efficiency of the machine will increase.

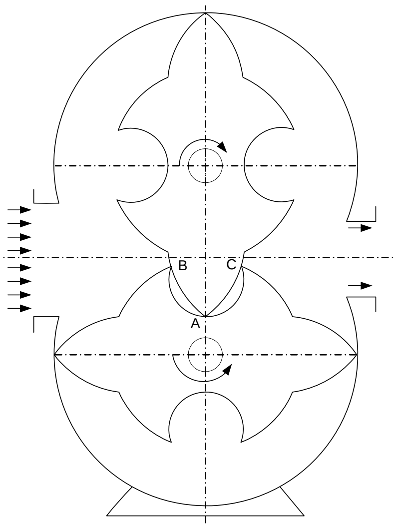

Fig. 9. Transversal section through the rotating machine.

3. CONCLUSIONS

The rotors design influences the flow rate, the power and the efficiency of the machine.

- For the rotor with the blade shaped (I), triangular (II) or curvilinear (III) piston, the flow rate decreases in the following order: I, II, III.
- The driving power of the machine will decrease in the same order: I, II, III.
- The volumetric efficiency will increase in the order I, II, III.
- In the I[st] version, between discharge and intake there is only one sealing zone (line) (A) on all the length of the rotor; accordingly, this solution is recommended for the construction of the pumps.
- In the III[rd] version, there are three contact zones B,A,C, on all the length of the rotor, providing a better sealing between discharge and intake; this solution is recommended for the construction of the high pressure ventilators and of low pressure compressors.

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